z.1 The Story in More Detail

sth:z:story: sec

In **??**, we quoted Schoenfield's description of the process of set-formation. We now want to write down a few more principles, to make this story a bit more precise. Here they are:

> *Stages-are-key.* Every set is formed at some stage.
>
> *Stages-are-ordered.* Stages are ordered: some come *before* others.[1]
>
> *Stages-accumulate.* For any stage S, and for any sets which were formed *before* stage S: a set is formed at stage S whose members are exactly those sets. Nothing else is formed at stage S.

These are informal principles, but we will be able to use them to vindicate several of the axioms of Zermelo's set theory.

(We should offer a word of caution. Although we will be presenting some completely standard axioms, with completely standard names, the italicised principles we have just presented have no particular names in the literature. I've just given them monikers which are hopefully helpful.)

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Bibliography

Scott, Dana. 1974. Axiomatizing set theory. In *Axiomatic Set Theory II*, ed. Thomas Jech, 207–14. American Mathematical Society. Proceedings of the Symposium in Pure Mathematics of the American Mathematical Society, July–August 1967.

[1] We will actually assume—tacitly—that the stages are *well-ordered*. What this amounts to is explained in **??**. This is a substantial assumption. In fact, using a very clever technique due to Scott (1974), this assumption can be *avoided* and then *derived*. (This will also explain why we should think that there is an initial stage.) But we cannot go into that here.