z.1 The Story in More Detail

In ??, we quoted Schoenfield’s description of the process of set-formation. We now want to write down a few more principles, to make this story a bit more precise. Here they are:

1. **Stages-are-key.** Every set is formed at some stage.
2. **Stages-are-ordered.** Stages are ordered: some come before others.\(^1\)
3. **Stages-accumulate.** For any stage \( S \), and for any sets which were formed before stage \( S \): a set is formed at stage \( S \) whose members are exactly those sets. Nothing else is formed at stage \( S \).

These are informal principles, but we will be able to use them to vindicate several of the axioms of Zermelo’s set theory.

(We should offer a word of caution. Although we will be presenting some completely standard axioms, with completely standard names, the italicised principles we have just presented have no particular names in the literature. I’ve just given them monikers which are hopefully helpful.)

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Bibliography


\(^1\)We will actually assume—tacitly—that the stages are well-ordered. What this amounts to is explained in ??, This is a substantial assumption. In fact, using a very clever technique due to Scott (1974), this assumption can be avoided and then derived. (This will also explain why we should think that there is an initial stage.) But we cannot go into that here.