

z.1 \mathbf{Z}^- : a Milestone

sth:z:milestone:
sec We will revisit *Stages-hit-infinity* in the next section. However, with the Axiom of Infinity, we have reached an important milestone. We now have all the axioms required for the theory \mathbf{Z}^- . In detail:

Definition z.1. The theory \mathbf{Z}^- has these axioms: Extensionality, Union, Pairs, Powersets, Infinity, and all instances of the Separation scheme.

The name stands for *Zermelo* set theory (*minus* something which we will come to later). Zermelo deserves the honour, since he essentially formulated this theory in his 1908.¹

This theory is powerful enough to allow us to do an enormous amount of mathematics. In particular, you *should* look back through ??, and convince yourself that everything we did, naïvely, could be done more formally within \mathbf{Z}^- . (Once you have done that for a bit, you might want to skip ahead and read ??.) So, henceforth, and without any further comment, we will take ourselves to be working in \mathbf{Z}^- (at least).

Photo Credits

Bibliography

Potter, Michael. 2004. *Set Theory and its Philosophy*. Oxford: Oxford University Press.

Zermelo, Ernst. 1908. Untersuchungen über die Grundlagen der Mengenlehre I. *Mathematische Annalen* 65: 261–81.

¹For interesting comments on the history and technicalities, see [Potter \(2004, Appendix A\)](#).