Stages-hit-infinity in the next section. However, with the Axiom of Infinity, we have reached an important milestone. We now have all the axioms required for the theory $Z^-$. In detail:

**Definition z.1.** The theory $Z^-$ has these axioms: Extensionality, Union, Pairs, Powersets, Infinity, and all instances of the Separation scheme.

The name stands for Zermelo set theory (*minus* something which we will come to later). Zermelo deserves the honour, since he essentially formulated this theory in his 1908.\(^1\)

This theory is powerful enough to allow us to do an enormous amount of mathematics. In particular, you *should* look back through ??, and convince yourself that everything we did, naively, could be done more formally within $Z^-$. (Once you have done that for a bit, you might want to skip ahead and read ??.) So, henceforth, and without any further comment, we will take ourselves to be working in $Z^-$ (at least).

**Photo Credits**

**Bibliography**


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\(^1\)For interesting comments on the history and technicalities, see Potter (2004, Appendix A).