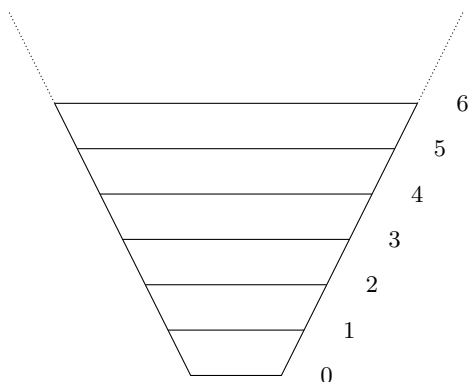


story.1 Urelements or Not?

sth:story:urelements:
sec

In the next few chapters, we will try to extract axioms from the cumulative-iterative conception of set. But, before going any further, we need to say something more about *urelements*.

The picture of ?? allowed us only to form new sets from old *sets*. However, we might want to allow that certain *non-sets*—cows, pigs, grains of sand, or whatever—can be *elements* of sets. In that case, we would start with certain basic elements, *urelements*, and then say that at each stage S we would form “all possible” sets consisting of urelements or sets formed at stages before S (in any combination). The resulting picture would look more like this:



So now we have a decision to take: *Should we allow urelements?*

Philosophically, it makes sense to include urelements in our theorising. The main reason for this is to make our set theory *applicable*. To illustrate the point, recall from ?? that we say that two sets A and B have the same size, i.e., $A \approx B$, iff there is a bijection between them. Now, if the cows in the field and the pigs in the sty both form sets, we can offer a set-theoretical treatment of the claim “there are as many cows as pigs”. But if we ban urelements, so that the cows and the pigs do *not* form sets, then that set-theoretical treatment will be unavailable. Indeed, we will have no straightforward ability to apply set theory to anything other than sets themselves. (For more reasons to include urelements, see [Potter 2004](#), pp. vi, 24, 50–1.)

Mathematically, however, it is quite rare to allow urelements. In part, this is because it is *very slightly* easier to formulate set theory without urelements. But, occasionally, one finds more interesting justifications for excluding urelement from set theory:

In accordance with the belief that set theory is the foundation of mathematics, we should be able to capture all of mathematics by just talking about sets, so our variable should not range over objects like cows and pigs. ([Kunen, 1980](#), p. 8)

So: a focus on applicability would suggest *including* urelements; a focus on a reductive foundational goal (reducing mathematics to pure set theory) might

suggest *excluding* them. Mild laziness, too, points in the direction of excluding urelements.

We will follow the laziest path. Partly, though, there is a pedagogical justification. Our aim is to introduce you to the elements of set theory that you would need in order to get started on the philosophy of set theory. And most of that philosophical literature discusses set theories formulated *without* urelements. So this book will, perhaps, be of more use, if it hews fairly closely to that literature.

Photo Credits

Bibliography

Kunen, Kenneth. 1980. *Set Theory: An Introduction to Independence Proofs*. New York: North Holland.

Potter, Michael. 2004. *Set Theory and its Philosophy*. Oxford: Oxford University Press.