syn.1 Semantic Notions

We define the following semantic notions:

Definition syn.1. 1. A formula $\varphi$ is satisfiable if for some $v$, $v \models \varphi$; it is unsatisfiable if for no $v$, $v \models \varphi$;
2. A formula $\varphi$ is a tautology if $v \models \varphi$ for all valuations $v$;
3. A formula $\varphi$ is contingent if it is satisfiable but not a tautology;
4. If $\Gamma$ is a set of formulas, $\Gamma \models \varphi$ ("$\Gamma$ entails $\varphi$") if and only if $v \models \varphi$ for every valuation $v$ for which $v \models \Gamma$.
5. If $\Gamma$ is a set of formulas, $\Gamma$ is satisfiable if there is a valuation $v$ for which $v \models \Gamma$, and $\Gamma$ is unsatisfiable otherwise.

Proposition syn.2.

1. $\varphi$ is a tautology if and only if $\emptyset \models \varphi$;
2. If $\Gamma \models \varphi$ and $\Gamma \models \varphi \rightarrow \psi$ then $\Gamma \models \psi$;
3. If $\Gamma$ is satisfiable then every finite subset of $\Gamma$ is also satisfiable;
4. Monotony: if $\Gamma \subseteq \Delta$ and $\Gamma \models \varphi$ then $\Delta \models \varphi$;
5. Transitivity: if $\Gamma \models \varphi$ and $\Delta \cup \{\varphi\} \models \psi$ then $\Gamma \cup \Delta \models \psi$;

Proof. Exercise.

Problem syn.1. Prove Proposition syn.2

Proposition syn.3. $\Gamma \models \varphi$ if and only if $\Gamma \cup \{\neg \varphi\}$ is unsatisfiable;

Proof. Exercise.

Problem syn.2. Prove Proposition syn.3

Theorem syn.4 (Semantic Deduction Theorem). $\Gamma \models \varphi \rightarrow \psi$ if and only if $\Gamma \cup \{\varphi\} \models \psi$.

Proof. Exercise.

Problem syn.3. Prove Theorem syn.4

Photo Credits

Bibliography