We define the following semantic notions:

**Definition syn.1.**
1. A formula \( \varphi \) is **satisfiable** if for some \( v \), \( v \models \varphi \); it is **unsatisfiable** if for no \( v \), \( v \models \varphi \);
2. A formula \( \varphi \) is a **tautology** if \( v \models \varphi \) for all valuations \( v \);
3. A formula \( \varphi \) is **contingent** if it is satisfiable but not a tautology;
4. If \( \Gamma \) is a set of formulas, \( \Gamma \models \varphi \) ("\( \Gamma \) entails \( \varphi \)") if and only if \( v \models \varphi \) for every valuation \( v \) for which \( v \models \Gamma \).
5. If \( \Gamma \) is a set of formulas, \( \Gamma \) is **satisfiable** if there is a valuation \( v \) for which \( v \models \Gamma \), and \( \Gamma \) is **unsatisfiable** otherwise.

**Proposition syn.2.**
1. \( \varphi \) is a tautology if and only if \( \emptyset \models \varphi \);
2. If \( \Gamma \models \varphi \) and \( \Gamma \models \varphi \rightarrow \psi \) then \( \Gamma \models \psi \);
3. If \( \Gamma \) is satisfiable then every finite subset of \( \Gamma \) is also satisfiable;
4. **Monotony:** if \( \Gamma \subseteq \Delta \) and \( \Gamma \models \varphi \) then also \( \Delta \models \varphi \);
5. **Transitivity:** if \( \Gamma \models \varphi \) and \( \Delta \cup \{ \varphi \} \models \psi \) then \( \Gamma \cup \Delta \models \psi \);

**Proof.** Exercise.

**Problem syn.1.** Prove Proposition syn.2

**Proposition syn.3.** \( \Gamma \models \varphi \) if and only if \( \Gamma \cup \{ \neg \varphi \} \) is unsatisfiable;

**Proof.** Exercise.

**Problem syn.2.** Prove Proposition syn.3

**Theorem syn.4 (Semantic Deduction Theorem).** \( \Gamma \models \varphi \rightarrow \psi \) if and only if \( \Gamma \cup \{ \varphi \} \models \psi \).

**Proof.** Exercise.

**Problem syn.3.** Prove Theorem syn.4

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Bibliography