## syn.1 Semantic Notions

pl:syn:sem:sec We define the following semantic notions:

**Definition syn.1.** 1. A formula $\varphi$ is *satisfiable* if for some $\mathfrak{v}$, $\mathfrak{v} \vDash \varphi$; it is *unsatisfiable* if for no $\mathfrak{v}$, $\mathfrak{v} \vDash \varphi$;

2. A formula $\varphi$ is a *tautology* if $\mathfrak{v} \vDash \varphi$ for all valuations $v$;

3. A formula $\varphi$ is *contingent* if it is satisfiable but not a tautology;

4. If $\Gamma$ is a set of formulas, $\Gamma \vDash \varphi$ ("$\Gamma$ entails $\varphi$") if and only if $\mathfrak{v} \vDash \varphi$ for every valuation $\mathfrak{v}$ for which $\mathfrak{v} \vDash \Gamma$.

5. If $\Gamma$ is a set of formulas, $\Gamma$ is *satisfiable* if there is a valuation $\mathfrak{v}$ for which $\mathfrak{v} \vDash \Gamma$, and $\Gamma$ is *unsatisfiable* otherwise.

*pl:syn:sem:prop:semanticalfacts* **Proposition syn.2.**

1. *$\varphi$ is a tautology if and only if $\emptyset \vDash \varphi$;*

2. *If $\Gamma \vDash \varphi$ and $\Gamma \vDash \varphi \rightarrow \psi$ then $\Gamma \vDash \psi$;*

3. *If $\Gamma$ is satisfiable then every finite subset of $\Gamma$ is also satisfiable;*

4. *pl:syn:sem:def:Monotony* *Monotony: if $\Gamma \subseteq \Delta$ and $\Gamma \vDash \varphi$ then also $\Delta \vDash \varphi$;*

5. *pl:syn:sem:def:Cut* *Transitivity: if $\Gamma \vDash \varphi$ and $\Delta \cup \{\varphi\} \vDash \psi$ then $\Gamma \cup \Delta \vDash \psi$;*

*Proof.* Exercise. □

**Problem syn.1.** Prove Proposition syn.2

*pl:syn:sem:prop:entails-unsat* **Proposition syn.3.** *$\Gamma \vDash \varphi$ if and only if $\Gamma \cup \{\neg\varphi\}$ is unsatisfiable;*

*Proof.* Exercise. □

**Problem syn.2.** Prove Proposition syn.3

*pl:syn:sem:thm:sem-deduction* **Theorem syn.4** (Semantic Deduction Theorem). *$\Gamma \vDash \varphi \rightarrow \psi$ if and only if $\Gamma \cup \{\varphi\} \vDash \psi$.*

*Proof.* Exercise. □

**Problem syn.3.** Prove Theorem syn.4

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## Bibliography