

syn.1 Semantic Notions

pl:syn:sem:
sec We define the following semantic notions:

- Definition syn.1.** 1. A formula φ is *satisfiable* if for some \mathbf{v} , $\mathbf{v} \models \varphi$; it is *unsatisfiable* if for no \mathbf{v} , $\mathbf{v} \models \varphi$;
2. A formula φ is a *tautology* if $\mathbf{v} \models \varphi$ for all valuations v ;
3. A formula φ is *contingent* if it is satisfiable but not a tautology;
4. If Γ is a set of formulas, $\Gamma \models \varphi$ (“ Γ entails φ ”) if and only if $\mathbf{v} \models \varphi$ for every valuation \mathbf{v} for which $\mathbf{v} \models \Gamma$.
5. If Γ is a set of formulas, Γ is *satisfiable* if there is a valuation \mathbf{v} for which $\mathbf{v} \models \Gamma$, and Γ is *unsatisfiable* otherwise.

pl:syn:sem:
prop:semanticalfacts

Proposition syn.2.

1. φ is a tautology if and only if $\emptyset \models \varphi$;
2. If $\Gamma \models \varphi$ and $\Gamma \models \varphi \rightarrow \psi$ then $\Gamma \models \psi$;
3. If Γ is satisfiable then every finite subset of Γ is also satisfiable;
4. *Monotony:* if $\Gamma \subseteq \Delta$ and $\Gamma \models \varphi$ then also $\Delta \models \varphi$;
5. *Transitivity:* if $\Gamma \models \varphi$ and $\Delta \cup \{\varphi\} \models \psi$ then $\Gamma \cup \Delta \models \psi$;

pl:syn:sem:
def:Monotony
pl:syn:sem:
def:Cut

Proof. Exercise. □

Problem syn.1. Prove Proposition syn.2

pl:syn:sem:
prop:entails-unsat

Proposition syn.3. $\Gamma \models \varphi$ if and only if $\Gamma \cup \{\neg\varphi\}$ is unsatisfiable;

Proof. Exercise. □

Problem syn.2. Prove Proposition syn.3

pl:syn:sem:
thm:sem-deduction

Theorem syn.4 (Semantic Deduction Theorem). $\Gamma \models \varphi \rightarrow \psi$ if and only if $\Gamma \cup \{\varphi\} \models \psi$.

Proof. Exercise. □

Problem syn.3. Prove Theorem syn.4

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Bibliography