$\dfrac{\sigma\,\mathbb{T}\,\Box\varphi}{\sigma\,\mathbb{T}\,\varphi}$ T$\Box$	$\dfrac{\sigma\,\mathbb{F}\,\Diamond\varphi}{\sigma\,\mathbb{F}\,\varphi}$ T$\Diamond$
$\dfrac{\sigma\,\mathbb{T}\,\Box\varphi}{\sigma\,\mathbb{T}\,\Diamond\varphi}$ D$\Box$	$\dfrac{\sigma\,\mathbb{F}\,\Diamond\varphi}{\sigma\,\mathbb{F}\,\Box\varphi}$ D$\Diamond$
$\dfrac{\sigma.n\,\mathbb{T}\,\Box\varphi}{\sigma\,\mathbb{T}\,\varphi}$ B$\Box$	$\dfrac{\sigma.n\,\mathbb{F}\,\Diamond\varphi}{\sigma\,\mathbb{F}\,\varphi}$ B$\Diamond$
$\dfrac{\sigma\,\mathbb{T}\,\Box\varphi}{\sigma.n\,\mathbb{T}\,\Box\varphi}$ 4$\Box$ $\sigma.n$ is used	$\dfrac{\sigma\,\mathbb{F}\,\Diamond\varphi}{\sigma.n\,\mathbb{F}\,\Diamond\varphi}$ 4$\Diamond$ $\sigma.n$ is used
$\dfrac{\sigma.n\,\mathbb{T}\,\Box\varphi}{\sigma\,\mathbb{T}\,\Box\varphi}$ 4r$\Box$	$\dfrac{\sigma.n\,\mathbb{F}\,\Diamond\varphi}{\sigma\,\mathbb{F}\,\Diamond\varphi}$ 4r$\Diamond$

Table 1: More modal rules.

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tab:more-rules

Logic	R is ...	Rules
T = KT	reflexive	T$\Box$, T$\Diamond$
D = KD	serial	D$\Box$, D$\Diamond$
K4	transitive	4$\Box$, 4$\Diamond$
B = KTB	reflexive, symmetric	T$\Box$, T$\Diamond$ B$\Box$, B$\Diamond$
S4 = KT4	reflexive, transitive	T$\Box$, T$\Diamond$, 4$\Box$, 4$\Diamond$
S5 = KT4B	reflexive, transitive, euclidean	T$\Box$, T$\Diamond$, 4$\Box$, 4$\Diamond$, 4r$\Box$, 4r$\Diamond$

Table 2: Tableau rules for various modal logics.

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tab:logics-rules

tab.1 Rules for Other Accessibility Relations

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sec

In order to deal with logics determined by special accessibility relations, we consider the additional rules in Table 1.

Adding these rules results in systems that are sound and complete for the logics given in Table 2.

Example tab.1. We give a closed tableau that shows $\mathbf{S5} \vdash 5$, i.e., $\Box\varphi \to \Box\Diamond\varphi$.

1.	$1\,\mathbb{F}\ \ \Box\varphi \to \Box\Diamond\varphi$	Assumption
2.	$1\,\mathbb{T}\ \ \Box\varphi$	$\to\mathbb{F}\,1$
3.	$1\,\mathbb{F}\ \ \Box\Diamond\varphi$	$\to\mathbb{F}\,1$
4.	$1.1\,\mathbb{F}\ \ \Diamond\varphi$	$\Box\mathbb{F}\,3$
5.	$1\,\mathbb{F}\ \ \Diamond\varphi$	$4\mathrm{r}\Diamond\ 4$
6.	$1.1\,\mathbb{F}\ \ \varphi$	$\Diamond\mathbb{F}\,5$
7.	$1.1\,\mathbb{T}\ \ \varphi$	$\Box\mathbb{T}\,2$
	$\otimes$	

Problem tab.1. Give closed tableaux that show the following:

1. **KT5** $\vdash$ B;
2. **KT5** $\vdash$ 4;
3. **KDB4** $\vdash$ T;
4. **KB4** $\vdash$ 5;
5. **KB5** $\vdash$ 4;
6. **KT** $\vdash$ D.

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Bibliography