Table 1: More modal rules.

<table>
<thead>
<tr>
<th>Logic</th>
<th>$R$ is ...</th>
<th>Rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbf{T} = \mathbf{KT}$</td>
<td>reflexive</td>
<td>$\sigma T \Box \varphi$, $\sigma T \Diamond$</td>
</tr>
<tr>
<td>$\mathbf{D} = \mathbf{KD}$</td>
<td>serial</td>
<td>$\sigma D \Box \varphi$, $\sigma D \Diamond$</td>
</tr>
<tr>
<td>$\mathbf{K4}$</td>
<td>transitive</td>
<td>$\sigma 4 \Box \varphi$, $\sigma 4 \Diamond$</td>
</tr>
<tr>
<td>$\mathbf{B} = \mathbf{KTB}$</td>
<td>reflexive, symmetric</td>
<td>$\sigma B \Box \varphi$, $\sigma B \Diamond$</td>
</tr>
<tr>
<td>$\mathbf{S4} = \mathbf{KT4}$</td>
<td>reflexive, transitive</td>
<td>$\sigma S4 \Box \varphi$, $\sigma S4 \Diamond$, $\sigma 4r \Box \varphi$, $\sigma 4r \Diamond$</td>
</tr>
<tr>
<td>$\mathbf{S5} = \mathbf{KT4B}$</td>
<td>reflexive, transitive, euclidean</td>
<td>$\sigma S5 \Box \varphi$, $\sigma S5 \Diamond$, $\sigma 4r \Box \varphi$, $\sigma 4r \Diamond$</td>
</tr>
</tbody>
</table>

Table 2: Tableau rules for various modal logics.

In order to deal with logics determined by special accessibility relations, we consider the additional rules in table 1. Adding these rules results in systems that are sound and complete for the logics given in table 2.
Example tab.1. We give a closed tableau that shows $S5 \vdash 5$, i.e., $\Box \varphi \rightarrow \Box \Diamond \varphi$.

1. 1F $\Box \varphi \rightarrow \Box \Diamond \varphi$ Assumption
2. 1T $\Box \varphi$ $\rightarrow$ F 1
3. 1F $\Box \Diamond \varphi$ $\rightarrow$ F 1
4. 1.1 F $\Diamond \varphi$ $\Box$ F 3
5. 1 F $\Diamond \varphi$ 4 $\Diamond$ 4
6. 1.1 F $\varphi$ $\Diamond$ F 5
7. 1.1 T $\varphi$ $\Box$ T 2

Problem tab.1. Give closed tableaux that show the following:

1. KT5 $\vdash$ B;
2. KT5 $\vdash$ 4;
3. KDB4 $\vdash$ T;
4. KB4 $\vdash$ 5;
5. KB5 $\vdash$ 4;
6. KT $\vdash$ D.

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Bibliography