syn.1 Truth in a Model

mod:syn:tru:sec

Definition syn.1. A formula φ is *true in a model* $M = \langle W, R, V \rangle$, written $\mathfrak{M} \models \varphi$, if and only if $\mathfrak{M}, w \models \varphi$ for every $w \in W$.

mod:syn:tru:prop:truthfacts

Proposition syn.2.

1. *If $\mathfrak{M} \models \varphi$ then $\mathfrak{M} \not\models \neg\varphi$, but* not *vice-versa.*

2. *If $\mathfrak{M} \models \varphi \rightarrow \psi$ then $\mathfrak{M} \models \varphi$ only if $\mathfrak{M} \models \psi$, but* not *vice-versa.*

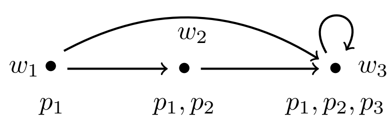
Proof.

1. If $\mathfrak{M} \models \varphi$ then φ is true at all worlds in W, and since $W \neq \emptyset$, it can't be that $\mathfrak{M} \models \neg\varphi$, or else φ would have to be both true and false at some world. Conversely, if $\mathfrak{M} \not\models \neg\varphi$ then φ is true at some world $w \in W$; it does not follow that $\mathfrak{M} \models \varphi$. For instance, in the model of **??**, $\mathfrak{M} \not\models \neg p$, but it does not follow that $\mathfrak{M} \models p$.

2. Assume $\mathfrak{M} \models \varphi \rightarrow \psi$ and $\mathfrak{M} \models \varphi$; to show $\mathfrak{M} \models \psi$ let $w \in W$ be an arbitrary world. Then $\mathfrak{M}, w \models \varphi \rightarrow \psi$ and $\mathfrak{M}, w \models \psi$, so $\mathfrak{M}, w \models \varphi$, and since w was arbitrary, $\mathfrak{M} \models \varphi$. The converse fails: we need to find a model $\mathfrak{M}$ such that $\mathfrak{M} \models \varphi$ only if $\mathfrak{M} \models \psi$, but $\mathfrak{M} \not\models \varphi \rightarrow \psi$. Consider again the model of **??**: $\mathfrak{M} \not\models p$ and hence (vacuously) $\mathfrak{M} \models p$ only if $\mathfrak{M} \models q$. However, $\mathfrak{M} \not\models p \rightarrow q$, as p is true but q false at w_1.

□

Problem syn.1. Consider the following model $\mathfrak{M}$ for the language comprising p_1, p_2, p_3 as the only propositional variables:



Are the following formulas and schemas true in the model $\mathfrak{M}$, i.e., true at every world in $\mathfrak{M}$? Explain.

1. $p \rightarrow \Diamond p$ (for p atomic);

2. $\varphi \rightarrow \Diamond \varphi$ (for φ arbitrary);

3. $\Box p \rightarrow p$ (for p atomic);

4. $\neg p \rightarrow \Diamond \Box p$ (for p atomic);

5. $\Diamond \Box \varphi$ (for φ arbitrary);

6. $\Box \Diamond p$ (for p atomic).

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Bibliography