syn.1 Truth at a World

mod:syn:trw:sec

mod:syn:trw:defn:mmodels **Definition syn.1.** *Truth of a formula φ at w* in a $\mathfrak{M}$, $\mathfrak{M}, w \models \varphi$, is defined inductively as follows:

1. $\mathfrak{M}, w \models p$ iff $w \in V(p)$
2. $\mathfrak{M}, w \models \top$
3. $\mathfrak{M}, w \not\models \bot$
4. $\mathfrak{M}, w \models \neg\psi$ iff $\mathfrak{M}, w \not\models \psi$
5. $\mathfrak{M}, w \models \varphi \land \psi$ iff $\mathfrak{M}, w \models \varphi$ and $\mathfrak{M}, w \models \psi$
6. $\mathfrak{M}, w \models \varphi \lor \psi$ iff $\mathfrak{M}, w \models \varphi$ or $\mathfrak{M}, w \models \psi$ (or both)
7. $\mathfrak{M}, w \models \varphi \to \psi$ iff $\mathfrak{M}, w \not\models \varphi$ or $\mathfrak{M}, w \models \psi$
8. mod:syn:trw:defn:sub:mmodels-box $\mathfrak{M}, w \models \Box\varphi$ iff $\mathfrak{M}, w' \models \varphi$ for all $w' \in W$ with wRw'

Note that by clause (8), a formula $\Box\psi$ is satisfied at w whenever there are no w' with wRw'. In such a case $\Box\psi$ is *vacuously* satisfied at w. Also, $\Box\psi$ may be satisifed at w even if ψ is not, and the truth of ψ at w does not guarantee the truth of $\Diamond\psi$ there—this holds if wRw, e.g., if R is reflexive.

Problem syn.1. Consider the model of **??**. Which of the following hold?

1. $\mathfrak{M}, w_1 \models q$;
2. $\mathfrak{M}, w_3 \models \neg q$;
3. $\mathfrak{M}, w_1 \models p \lor q$;
4. $\mathfrak{M}, w_1 \models \Box(p \lor q)$;
5. $\mathfrak{M}, w_3 \models \Box q$;
6. $\mathfrak{M}, w_3 \models \Box\bot$;
7. $\mathfrak{M}, w_1 \models \Diamond q$;
8. $\mathfrak{M}, w_1 \models \Box q$;
9. $\mathfrak{M}, w_1 \models \neg\Box\Box\neg q$.

Problem syn.2. Let $\mathfrak{M} = \langle W, R, V\rangle$ be a model, and suppose $u, v \in W$ are such that:

1. $u \in V(p)$ if and only if $v \in V(p)$; and
2. for all $w \in W$: Ruw if and only if Rvw.

Using induction on formulas, show that for all formulas φ: $\mathfrak{M}, u \models \varphi$ if and only if $\mathfrak{M}, v \models \varphi$.

Problem syn.3. Let $\mathfrak{M} = \langle M, R, V \rangle$. Show that $\mathfrak{M}, w \models \Diamond\varphi$ if and only if, for some w' with Rww', $\mathfrak{M}, w' \models \varphi]$.

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Bibliography