syn.1 Simultaneous Substitution

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Definition syn.1. Where φ is a modal formula all of whose propositional variables are among p_1, …, p_n, and χ_1, …, χ_n are also modal formulas, we define $\varphi[\chi_1/p_1, \dots, \chi_n/p_n]$ as the result of simultaneously substituting each χ_i for p_i in A. Formally, this is a definition by induction on φ:

1. If φ is the propositional variable q, then $\varphi[\chi_1/p_1, \dots, \chi_n/p_n] = \chi_i$ if Q is p_i for some $i \leq n$, and $\varphi[\chi_1/p_1, \dots, \chi_n/p_n] = Q$ otherwise.
2. If $A = \neg\psi$, then $\varphi[\chi_1/p_1, \dots, \chi_n/p_n] = \neg\psi[\chi_1/p_1, \dots, \chi_n/p_n]$.
3. If $\varphi = (\psi \land \theta)$, then
$$\varphi[\chi_1/p_1, \dots, \chi_n/p_n] = (\psi[\chi_1/p_1, \dots, \chi_n/p_n] \land \theta[\chi_1/p_1, \dots, \chi_n/p_n]).$$
4. If $\varphi = (\psi \lor \theta)$, then
$$\varphi[\chi_1/p_1, \dots, \chi_n/p_n] = (\psi[\chi_1/p_1, \dots, \chi_n/p_n] \lor \theta[\chi_1/p_1, \dots, \chi_n/p_n]).$$
5. If $\varphi = (\psi \to \theta)$, then
$$\varphi[\chi_1/p_1, \dots, \chi_n/p_n] = (\psi[\chi_1/p_1, \dots, \chi_n/p_n] \to \theta[\chi_1/p_1, \dots, \chi_n/p_n]).$$
6. If $\varphi = (\psi \leftrightarrow \theta)$, then
$$\varphi[\chi_1/p_1, \dots, \chi_n/p_n] = (\psi[\chi_1/p_1, \dots, \chi_n/p_n] \leftrightarrow \theta[\chi_1/p_1, \dots, \chi_n/p_n]).$$
7. If $\varphi = \Box\psi$, then $\varphi[\chi_1/p_1, \dots, \chi_n/p_n] = \Box\psi[\chi_1/p_1, \dots, \chi_n/p_n]$.
8. If $\varphi = \Diamond\psi$, then $\varphi[\chi_1/p_1, \dots, \chi_n/p_n] = \Diamond\psi[\chi_1/p_1, \dots, \chi_n/p_n]$.

The formula $\varphi[\chi_1/p_1, \dots, \chi_n/p_n]$ is called a *substitution instance* of φ.

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Bibliography