The basic concept of semantics for normal modal logics is that of a relational model. It consists of a set of worlds, which are related by a binary “accessibility relation,” together with an assignment which determines which propositional variables count as “true” at which worlds.

Definition syn.1. A model for the basic modal language is a triple $M = \langle W, R, V \rangle$, where

1. $W$ is a nonempty set of “worlds,”
2. $R$ is a binary accessibility relation on $W$, and
3. $V$ is a function assigning to each propositional variable $p$ a set $V(p)$ of possible worlds.

When $Rw w'$ holds, we say that $w'$ is accessible from $w$. When $w \in V(p)$ we say $p$ is true at $w$.

The great advantage of relational semantics is that models can be represented by means of simple diagrams, such as the one in Figure 1. Worlds are represented by nodes, and world $w'$ is accessible from $w$ precisely when there is an arrow from $w$ to $w'$. Moreover, we label a node (world) by $p$ when $w \in V(p)$, and otherwise by $\neg p$. Figure 1 represents the model with $W = \{w_1, w_2, w_3\}$, $R = \{(w_1, w_2), (w_1, w_3)\}$, $V(p) = \{w_1, w_2\}$, and $V(q) = \{w_2\}$.

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Bibliography