

## syn.1 Validity

mod:syn:val:  
sec

**Definition syn.1.** A formula  $\varphi$  is *valid* in a class  $\mathcal{C}$  of models if it is true in every model in  $\mathcal{C}$  (i.e., true at every world in every model in  $\mathcal{C}$ ). If  $\varphi$  is valid in  $\mathcal{C}$  we write  $\mathcal{C} \models \varphi$ , and we write  $\models \varphi$  if  $\varphi$  is valid in the class of *all* models.

**Proposition syn.2.** If  $\varphi$  is valid in  $\mathcal{C}$  it is also valid in each class  $\mathcal{C}' \subseteq \mathcal{C}$ .

mod:syn:val:  
prop:Nec-rule

**Proposition syn.3.** If  $\varphi$  is valid, then so is  $\Box\varphi$ .

*Proof.* Assume  $\models \varphi$ . To show  $\models \Box\varphi$  let  $\mathfrak{M} = \langle W, R, V \rangle$  be a model and  $w \in W$ . If  $Rww'$  then  $\mathfrak{M}, w' \models \varphi$ , since  $\varphi$  is valid, and so also  $\mathfrak{M}, w \models \Box\varphi$ . Since  $\mathfrak{M}$  and  $w$  were arbitrary,  $\models \Box\varphi$ .  $\square$

**Problem syn.1.** Show that the following are valid:

1.  $\models \Box p \rightarrow \Box(q \rightarrow p)$ ;
2.  $\models \Box \neg \perp$ ;
3.  $\models \Box p \rightarrow (\Box q \rightarrow \Box p)$ .

**Problem syn.2.** Show that  $\varphi \rightarrow \Box\varphi$  is valid in the class  $\mathcal{C}$  of models  $\mathfrak{M} = \langle W, R, V \rangle$  where  $W = \{w\}$ . Similarly, show that  $\psi \rightarrow \Box\psi$  and  $\Diamond\varphi \rightarrow \psi$  are valid in the class of models  $\mathfrak{M} = \langle W, R, V \rangle$  where  $R = \emptyset$ .

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## Bibliography