### syn.1 Validity

**Formulas** that are true in all models, i.e., true at every world in every model, are particularly interesting. They represent those modal propositions which are true regardless of how □ and ♦ are interpreted, as long as the interpretation is “normal” in the sense that it is generated by some accessibility relation on possible worlds. We call such **formulas valid**. For instance, □(p ∧ q) → □p is valid. Some **formulas** one might expect to be valid on the basis of the alethic interpretation of □, such as □p → p, are not valid, however. Part of the interest of relational models is that different interpretations of □ and ♦ can be captured by different kinds of accessibility relations. This suggests that we should define validity not just relative to all models, but relative to all models of a certain kind. It will turn out, e.g., that □p → p is true in all models where every world is accessible from itself, i.e., R is reflexive. Defining validity relative to classes of models enables us to formulate this succinctly: □p → p is valid in the class of reflexive models.

**Definition syn.1.** A formula ϕ is **valid** in a class C of models if it is true in every model in C (i.e., true at every world in every model in C). If ϕ is valid in C, we write C ⊨ ϕ, and we write ⊨ ϕ if ϕ is valid in the class of all models.

**Proposition syn.2.** If ϕ is valid in C it is also valid in each class C′ ⊆ C.

**Proposition syn.3.** If ϕ is valid, then so is □ϕ.

**Proof.** Assume ⊨ ϕ. To show ⊨ □ϕ let M = ⟨W, R, V⟩ be a model and w ∈ W. If Rw w′ then M, w′ ⊨ ϕ, since ϕ is valid, and so also M, w ⊨ □ϕ. Since M and w were arbitrary, ⊨ □ϕ.

**Problem syn.1.** Show that the following are valid:

1. ⊨ □p → □(q → p);
2. ⊨ □¬⊥;
3. ⊨ □p → (□q → □p).

**Problem syn.2.** Show that ϕ → □ϕ is valid in the class C of models M = ⟨W, R, V⟩ where W = {w}. Similarly, show that ψ → □ψ and ♦ϕ → ψ are valid in the class of models M = ⟨W, R, V⟩ where R = ∅.

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