

syn.1 The Language of Basic Modal Logic

mod:syn:lan:
sec The basic language of modal logic contains a set Var of **propositional variables** p_1, p_2, \dots , the familiar logical connectives \neg (“not”), \wedge (“and”), \vee (“or”), \rightarrow , (“if . . . then”), the symbols \top (the truth symbol) and \perp (the falsity symbol), as well as the two basic modalities \Box and \Diamond .

Definition syn.1. *Formulas* of the basic modal language are inductively defined as follows:

1. Every propositional variable p_i is an (atomic) **formula**.
2. \top is an (atomic) **formula**
3. \perp is an (atomic) **formula**.
4. If φ is a formula, so is $\neg\varphi$.
5. If φ and ψ are formulas, so are $(\varphi \wedge \psi)$, $(\varphi \vee \psi)$, $(\varphi \rightarrow \psi)$, and $(\varphi \leftrightarrow \psi)$.
6. If φ is a formula, so is $\Box\varphi$.
7. Nothing else is a **formula**.

If a **formula** φ does not contain \Box , we say it is *modal-free*.

$\Diamond A$ abbreviates $\neg\Box\neg\varphi$. So for instance, $\Diamond\Box p \rightarrow \Diamond\Diamond p$ is short for $\neg\Box\neg\Box p \rightarrow \neg\Box\neg\neg\Box p$.

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Bibliography