seq.1 Sequent Derivations for K

nml:seq:prk:sec

Example seq.1. We give a sequent calculus derivation that shows $\vdash (\Box\varphi \wedge \Box\psi) \to \Box(\varphi \wedge \psi)$.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\varphi \Rightarrow \varphi}{\psi, \varphi \Rightarrow \varphi}\,\mathrm{W} \quad \dfrac{\psi \Rightarrow \psi}{\psi, \varphi \Rightarrow \psi}\,\mathrm{W}}{\psi, \varphi \Rightarrow \varphi \wedge \psi}\,\wedge\mathrm{R}}{\Box\psi, \Box\varphi \Rightarrow \Box(\varphi \wedge \psi)}\,\Box}{\Box\varphi \wedge \Box\psi, \Box\varphi \Rightarrow \Box(\varphi \wedge \psi)}\,\wedge\mathrm{L}}{\Box\varphi, \Box\varphi \wedge \Box\psi \Rightarrow \Box(\varphi \wedge \psi)}\,\mathrm{XL}}{\Box\varphi \wedge \Box\psi, \Box\varphi \wedge \Box\psi \Rightarrow \Box(\varphi \wedge \psi)}\,\wedge\mathrm{L}}{\Box\varphi \wedge \Box\psi \Rightarrow \Box(\varphi \wedge \psi)}\,\mathrm{CL}}{\Rightarrow (\Box\varphi \wedge \Box\psi) \to \Box(\varphi \wedge \psi)}\,\to\mathrm{R}$$

Example seq.2. We give a sequent calculus derivation that shows $\vdash \Diamond(\varphi \vee \psi) \to (\Diamond\varphi \vee \Diamond\psi)$.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\varphi \Rightarrow \varphi}{\varphi \Rightarrow \varphi, \psi}\,\mathrm{W} \quad \dfrac{\psi \Rightarrow \psi}{\psi \Rightarrow \varphi, \psi}\,\mathrm{W}}{\varphi \vee \psi \Rightarrow \varphi, \psi}\,\vee\mathrm{L}}{\Diamond(\varphi \vee \psi) \Rightarrow \Diamond\varphi, \Diamond\psi}\,\Diamond}{\Diamond(\varphi \vee \psi) \Rightarrow \Diamond\varphi, \Diamond\varphi \vee \Diamond\psi}\,\vee\mathrm{R}}{\Diamond(\varphi \vee \psi) \Rightarrow \Diamond\varphi \vee \Diamond\psi, \Diamond\varphi}\,\mathrm{XR}}{\Diamond(\varphi \vee \psi) \Rightarrow \Diamond\varphi \vee \Diamond\psi, \Diamond\varphi \vee \Diamond\psi}\,\vee\mathrm{R}}{\Diamond(\varphi \vee \psi) \Rightarrow \Diamond\varphi \vee \Diamond\psi}\,\mathrm{CR}}{\Rightarrow \Diamond(\varphi \vee \psi) \to (\Diamond\varphi \vee \Diamond\psi)}\,\to\mathrm{R}$$

Here is a derivation of DUAL.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\varphi \Rightarrow \varphi}{\neg\varphi, \varphi \Rightarrow}\,\neg\mathrm{R}}{\Diamond\neg\varphi, \Box\varphi \Rightarrow}\,\Diamond}{\Box\varphi \Rightarrow \neg\Diamond\neg\varphi}\,\neg\mathrm{R}}{\Rightarrow \Box\varphi \to \neg\Diamond\neg\varphi}\,\to\mathrm{R} \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\varphi \Rightarrow \varphi}{\Rightarrow \varphi, \neg\varphi}\,\neg\mathrm{R}}{\Rightarrow \neg\varphi, \varphi}\,\mathrm{XR}}{\Rightarrow \Diamond\neg\varphi, \Box\varphi}\,\Box}{\Rightarrow \Box\varphi, \Diamond\neg\varphi}\,\mathrm{XR}}{\neg\Diamond\neg\varphi \Rightarrow \Box\varphi}\,\neg\mathrm{R}}{\Rightarrow \neg\Diamond\neg\varphi \to \Box\varphi}\,\to\mathrm{R}}{\Rightarrow \Box\varphi \leftrightarrow \neg\Diamond\neg\varphi}\,\wedge\mathrm{R}$$

Problem seq.1. Find sequent calculus proofs in **K** for the following formulas:

1. $\Box\neg p \to \Box(p \to q)$
2. $(\Box p \vee \Box q) \to \Box(p \vee q)$

3. $\Diamond p \to \Diamond(p \vee q)$
4. $\Box(p \wedge q) \to \Box p$

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Bibliography