

frd.1 Introduction

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One question that interests modal logicians is the relationship between the accessibility relation and the truth of certain **formulas** in models with that accessibility relation. For instance, suppose the accessibility relation is reflexive, i.e., for every $w \in W$, Rww . In other words, every world is accessible from itself. That means that when $\Box\varphi$ is true at a world w , w itself is among the accessible worlds at which φ must therefore be true. So, if the accessibility relation R of \mathfrak{M} is reflexive, then whatever world w and formula φ we take, $\Box\varphi \rightarrow \varphi$ will be true there (in other words, the schema $\Box p \rightarrow p$ and all its substitution instances are true in \mathfrak{M}).

The converse, however, is false. It's not the case, e.g., that if $\Box p \rightarrow p$ is true in \mathfrak{M} , then R is reflexive. For we can easily find a non-reflexive model \mathfrak{M} where $\Box p \rightarrow p$ is true at all worlds: take the model with a single world w , not accessible from itself, but with $w \in V(p)$. By picking the truth value of p suitably, we can make $\Box\varphi \rightarrow \varphi$ true in a model that is not reflexive.

The solution is to remove the variable assignment V from the equation. If we require that $\Box p \rightarrow p$ is true at all worlds in \mathfrak{M} , regardless of which worlds are in $V(p)$, then it is necessary that R is reflexive. For in any non-reflexive model, there will be at least one world w such that not Rww . If we set $V(p) = W \setminus \{w\}$, then p will be true at all worlds other than w , and so at all worlds accessible from w (since w is guaranteed not to be accessible from w , and w is the only world where p is false). On the other hand, p is false at w , so $\Box p \rightarrow p$ is false at w .

This suggests that we should introduce a notation for model structures without a valuation: we call these *frames*. A frame \mathfrak{F} is simply a pair $\langle W, R \rangle$ consisting of a set of worlds with an accessibility relation. Every model $\langle W, R, V \rangle$ is then, as we say, *based on* the frame $\langle W, R \rangle$. Conversely, a frame determines the class of models based on it; and a class of frames determines the class of models which are based on any frame in the class. And we can define $\mathfrak{F} \Vdash \varphi$, the notion of a **formula** being *valid* in a frame as: $\mathfrak{M} \Vdash \varphi$ for all \mathfrak{M} based on \mathfrak{F} .

With this notation, we can establish correspondence relations between **formulas** and classes of frames: e.g., $\mathfrak{F} \Vdash \Box p \rightarrow p$ if, and only if, \mathfrak{F} is reflexive.

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Bibliography