frd.1 Frames

Definition frd.1. A frame is a pair $\mathcal{F} = (W, R)$ where $W$ is a non-empty set of worlds and $R$ a binary relation on $W$. A model $\mathcal{M}$ is based on a frame $\mathcal{F} = (W, R)\) if and only if $\mathcal{M} = (W, R, V)$.

Definition frd.2. If $\mathcal{F}$ is a frame, we say that $\varphi$ is valid in $\mathcal{F}$, $\mathcal{F} \vDash \varphi$, if $\mathcal{M} \vDash \varphi$ for every model $\mathcal{M}$ based on $\mathcal{F}$.

If $\mathcal{F}$ is a class of frames, we say $\varphi$ is valid in $\mathcal{F}$, $\mathcal{F} \vDash \varphi$, if $\mathcal{F} \vDash \varphi$ for every frame $\mathcal{F} \in \mathcal{F}$.

The reason frames are interesting is that correspondence between schemas and properties of the accessibility relation $R$ is at the level of frames, not of models. For instance, although $T$ is true in all reflexive models, not every model in which $T$ is true is reflexive. However, it is true that not only is $T$ valid on all reflexive frames, also every frame in which $T$ is valid is reflexive.

Remark 1. Validity in a class of frames is a special case of the notion of validity in a class of models: $\mathcal{F} \vDash \varphi$ iff $\mathcal{C} \vDash \varphi$ where $\mathcal{C}$ is the class of all models based on a frame in $\mathcal{F}$.

Obviously, if a formula or a schema is valid, i.e., valid with respect to the class of all models, it is also valid with respect to any class $\mathcal{F}$ of frames.

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Bibliography