frd.1 Frames

mod:frd:fra:sec

Definition frd.1. A *frame* is a pair $\mathfrak{F} = \langle W, R \rangle$ where W is a non-empty set of worlds and R a binary relation on W. A model $\mathfrak{M}$ is *based on* a frame $\mathfrak{F} = \langle W, R \rangle$ if and only if $\mathfrak{M} = \langle W, R, V \rangle$.

Definition frd.2. If $\mathfrak{F}$ is a frame, we say that φ is *valid in* $\mathfrak{F}$, $\mathfrak{F} \vDash \varphi$, if $\mathfrak{M} \Vdash \varphi$ for every model $\mathfrak{M}$ based on $\mathfrak{F}$.

If $\mathcal{F}$ is a class of frames, we say φ is *valid in* $\mathcal{F}$, $\mathcal{F} \vDash \varphi$, iff $\mathfrak{F} \vDash \varphi$ for every frame $\mathfrak{F} \in \mathcal{F}$.

The reason frames are interesting is that correspondence between schemas and properties of the accessibility relation R is at the level of frames, *not of models*. For instance, although T is true in all reflexive models, not every model in which T is true is reflexive. However, it *is* true that not only is T *valid* on all reflexive *frames*, also every frame in which T is valid is reflexive.

Remark 1. Validity in a class of frames is a special case of the notion of validity in a class of models: $\mathcal{F} \vDash \varphi$ iff $\mathcal{C} \vDash \varphi$ where $\mathcal{C}$ is the class of all models based on a frame in $\mathcal{F}$.

Obviously, if a formula or a schema is valid, i.e., valid with respect to the class of *all* models, it is also valid with respect to any class $\mathcal{F}$ of frames.

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Bibliography