fil.1 Preliminaries

mod:fil:pre:sec

Filtrations allow us to establish the decidability of our systems of modal logic by showing that they have the *finite model property*, i.e., that any formula that is true (false) in a model is also true (false) in a *finite* model.

mod:fil:pre:def:modallyclosed

Definition fil.1. A set Γ of formulas is *closed under subformulas* if it contains every subformula of a formula in Γ. Further, Γ is *modally closed* if it is closed under subformulas and moreover $\varphi \in \Gamma$ implies $\Box\varphi, \Diamond\varphi \in \Gamma$.

Definition fil.2. Let $\mathfrak{M} = \langle W, R, V \rangle$ and suppose Γ is closed under subformulas. Define a relation $\equiv$ on W to hold of any two worlds that make true the same formulas from Γ, i.e.:

$$u \equiv v \quad \text{if and only if} \quad \forall \varphi \in \Gamma : \mathfrak{M}, u \models \varphi \Leftrightarrow \mathfrak{N}, v \models \varphi.$$

Clearly, $\equiv$ is an equivalence relation over W. Standardly, for any $w \in W$, the equivalence class of w is denoted by $[w]$.

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Bibliography