

fl.1 Filtrations are Finite

mod:fil:fin:
sec We've defined filtrations for any set Γ that is closed under sub-formulas. Nothing in the definition itself guarantees that filtrations are finite. In fact, when Γ is infinite (e.g., is the set of all formulas), it may well be infinite. However, if Γ is finite (e.g., when it is the set of sub-formulas of a given formula φ), so is any filtration through Γ .

mod:fil:fin:
prop:fil-are-finite **Proposition fl.1.** *If Γ is finite then any filtration \mathfrak{M}^* of a model \mathfrak{M} through Γ is also finite.*

Proof. The size of W^* is the number of different classes $[w]$ under the equivalence relation \equiv . Any two worlds u, v in such class—that is, any u and v such that $u \equiv v$ —agree on all formulas φ in Γ , $\varphi \in \Gamma$ either φ is true at both u and v , or at neither. So each class $[w]$ corresponds to subset of Γ , namely the set of all $\varphi \in \Gamma$ such that φ is true at the worlds in $[w]$. No two different classes $[u]$ and $[v]$ correspond to the same subset of Γ . For if the set of formulas true at u and that of formulas true at v are the same, then u and v agree on all formulas in Γ , i.e., $u \equiv v$. But then $[u] = [v]$. So, there is an injective function from W^* to $\wp(\Gamma)$, and hence $|W^*| \leq |\wp(\Gamma)|$. Hence if Γ contains n sentences, the cardinality of W^* is no greater than 2^n . \square

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Bibliography