fil.1 Filtrations of Euclidean Models

mod:fil:euc:sec The approach of **??** does not work in the case of models that are euclidean or serial and euclidean. Consider the model at the top of Figure 1, which is both euclidean and serial. Let $\Gamma = \{p, \Box p\}$. When taking a filtration through Γ, then $[w_1] = [w_3]$ since w_1 and w_3 are the only worlds that agree on Γ. Any filtration will also have the arrow inherited from $\mathfrak{M}$, as depicted in Figure 2. But we cannot add arrows to that model in order to make it euclidean, for then there would be a double arrow between w_2 and w_4, and hence also between w_2 and w_5. But $\Box p$ is true at w_2 while p is false at w_5.

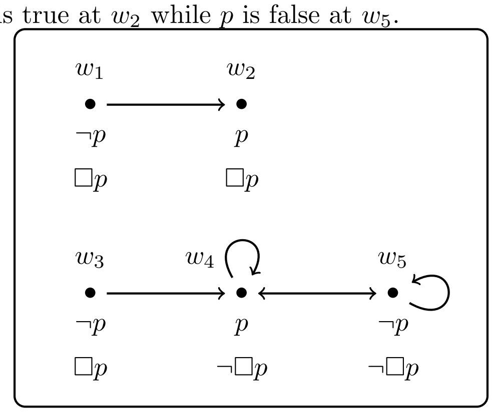

Figure 1: A serial and euclidean model.

mod:fil:euc:fig:ser-eucl

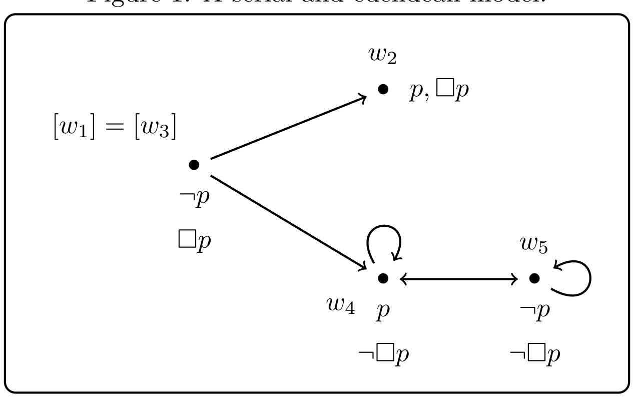

Figure 2: The filtration of the model in Figure 1.

mod:fil:euc:fig:ser-eucl2 In particular, it is not enough to consider filtrations through arbitrary Γ's closed under subsentences. Instead we need to consider sets Γ that are *modally closed* (see **??**). Such sets of sentences are infinite, and therefore do not lead immediately to the decidability of the corresponding system.

Theorem fil.1. *Let Γ be modally closed and $\mathfrak{M} = \langle W, R, V\rangle$. If $\mathfrak{M}^* = \langle W^*, R^*, V^*\rangle$ is a coarsest filtration of $\mathfrak{M}$, then $\mathfrak{M}^*$ is symmetric, transitive or euclidean if $\mathfrak{M}$ is symmetric, transitive, or euclidean, respectively.*

Proof. The proof of transitivity uses the validity of both 4 and $4_\Diamond$ in all transitive models, and likewise euclideanness uses the fact that both 5 and $5_\Diamond$ are valid in all euclidean models, and the proof of symmetry likewise uses both B and $\mathrm{B}_\Diamond$.

If $\mathfrak{M}^*$ is a coarsest filtration, then by definition $R^*[u][v]$ holds if and only if $C_1(u,v)$. For transitivity, suppose $C_1(u,v)$ and $C_1(v,w)$: to show $C_1(u,w)$ suppose $\mathfrak{M}, u \models \Box\varphi$; then $\mathfrak{M}, u \models \Box\Box\varphi$; since $\Box\Box\varphi \in \Gamma$ by closure, also by $C_1(u,v)$, $\mathfrak{M}, v \models \Box\varphi$ and by $C_1(v,w)$, also $\mathfrak{M}, w \models \varphi$. The case for $\Diamond\varphi$ is similar. $\Box$

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Bibliography