fil.1 S5 has the Finite Model Property

mod:fil:fmp:sec

Definition fil.1. A system Σ of modal logic is said to have the *finite model property* if whenever a formula φ is true at a world in a model of Σ then φ is true at a world in a *finite* model of Σ.

mod:fil:fmp:prop:univ-fin **Proposition fil.2.** *Let $\mathcal{U}$ be the class of universal models (see* **??***) and $\mathcal{U}_{\mathrm{Fin}}$ the class of all finite universal models. Then any formula φ is valid in $\mathcal{U}$ if and only if it is valid in $\mathcal{U}_{\mathrm{Fin}}$.*

Proof. Finite universal models are universal models, so the left-to-right direction is trivial. For the right-to left direction, suppose that φ is false at some world w in a universal model $\mathfrak{M}$. Let Γ contain φ as well as all of its subformulas; clearly Γ is finite. Take a filtration $\mathfrak{M}^*$ of $\mathfrak{M}$; then $\mathfrak{M}^*$ is finite by **??**, and by **??**, φ is false at $[w]$ in $\mathfrak{M}^*$. It remains to observe that $\mathfrak{M}^*$ is also universal: given u and v, by hypothesis Ruv and by Definition **????**, also $R^*[u][v]$. □

mod:fil:fmp:cor:S5fmp **Corollary fil.3.** **S5** *has the finite model property.*

Proof. By **??** and Proposition fil.2, if φ is true at a world in some reflexive and euclidean model then it is true at a world in a finite universal model (universal models are obviously reflexive and euclidean). □

Problem fil.1. Show that any filtration of a serial or reflexive model is also serial or reflexive (respectively).

Problem fil.2. Find a non-symmetric (non-transitive, non-euclidean) filtration of a symmetric (transitive, euclidean) model.

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Bibliography