S5 is Decidable

The finite model property gives us an easy way to show that systems of modal logic given by schemas are decidable (i.e., that there is a computable procedure to determine whether a formula is derivable in the system or not).

**Theorem fil.1. S5 is decidable.**

**Proof.** Let $\varphi$ be given, and suppose the propositional variables occurring in $\varphi$ are among $p_1, \ldots, p_k$. Since for each $n$ there are only finitely many models with $n$ worlds assigning a value to $p_1, \ldots, p_k$, we can enumerate, in parallel, all the theorems of S5 by generating proofs in some systematic way; and all the models containing 1, 2, \ldots worlds and checking whether $\varphi$ fails at a world in some such model. Eventually one of the two parallel processes will give an answer, as by ?? and ??, either $\varphi$ is derivable or it fails in a finite universal model.

The above proof works for S5 because filtrations of universal models are automatically universal. The same holds for reflexivity and seriality, but more work is needed for other properties.

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**Bibliography**