

## fil.1 S5 is Decidable

mod:fil:dec:  
sec The finite model property gives us an easy way to show that systems of modal logic given by schemas are *decidable* (i.e., that there is a computable procedure to determine whether a **formulas** is **derivable** in the system or not).

**Theorem fil.1.** *S5 is decidable.*

*Proof.* Let  $\varphi$  be given, and suppose the propositional variables occurring in  $\varphi$  are among  $p_1, \dots, p_k$ . Since for each  $n$  there are only finitely many models with  $n$  worlds assigning a value to  $p_1, \dots, p_k$ , we can enumerate, *in parallel*, all the theorems of **S5** by generating proofs in some systematic way; and all the models containing 1, 2,  $\dots$  worlds and checking whether  $\varphi$  fails at a world in some such model. Eventually one of the two parallel processes will give an answer, as by ?? and ??, either  $\varphi$  is **derivable** or it fails in a finite universal model.  $\square$

The above proof works for **S5** because filtrations of universal models are automatically universal. The same holds for reflexivity and seriality, but more work is needed for other properties.

**Problem fil.1.** Show that any filtration of a serial or reflexive model is also serial or reflexive (respectively).

**Problem fil.2.** Find a non-symmetric (non-transitive, non-euclidean) filtration of a symmetric (transitive, euclidean) model.

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## Bibliography