

fil.1 S5 is Decidable

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sec

The finite model property gives us an easy way to show that systems of modal logic given by schemas are *decidable* (i.e., that there is a computable procedure to determine whether a formulas is *derivable* in the system or not).

Theorem fil.1. *S5 is decidable.*

Proof. Let φ be given, and suppose the propositional variables occurring in φ are among p_1, \dots, p_k . Since for each n there are only finitely many models with n worlds assigning a value to p_1, \dots, p_k , we can enumerate, *in parallel*, all the theorems of **S5** by generating proofs in some systematic way; and all the models containing 1, 2, \dots worlds and checking whether φ fails at a world in some such model. Eventually one of the two parallel processes will give an answer, as by ?? and ??, either φ is *derivable* or it fails in a finite universal model. \square

The above proof works for **S5** because filtrations of universal models are automatically universal. The same holds for reflexivity and seriality, but more work is needed for other properties.

Problem fil.1. Show that any filtration of a serial or reflexive model is also serial or reflexive (respectively).

Problem fil.2. Find a non-symmetric (non-transitive, non-euclidean) filtration of a symmetric (transitive, euclidean) model.

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Bibliography