

com.1 Derivability and Complete Consistent Sets

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sec

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cor:provability-characterization

Corollary com.1. $\Gamma \vdash_{\Sigma} \varphi$ if and only if $\varphi \in \Delta$ for each complete Σ -consistent set Δ extending Γ (including when $\Gamma = \emptyset$, in which case we get another characterization of the modal system Σ .)

Proof. Suppose $\Gamma \vdash_{\Sigma} \varphi$, and let Δ be any complete Σ -consistent set extending Γ . If $\varphi \notin \Delta$ then by maximality $\neg\varphi \in \Delta$ and so $\Delta \vdash_{\Sigma} \varphi$ (by monotony) and $\Delta \vdash_{\Sigma} \neg\varphi$ (by reflexivity), and so Δ is inconsistent. Conversely if $\Gamma \not\vdash_{\Sigma} \varphi$, then $\Gamma \cup \{\neg\varphi\}$ is Σ -consistent, and by Lindenbaum's Lemma there is a complete consistent set Δ extending $\Gamma \cup \{\neg\varphi\}$. By consistency, $\varphi \notin \Delta$. \square

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Bibliography