com.1 Completeness for K

mod:com:cmk:
sec

mod:com:cmk:
thm:determination

Theorem com.1 (Determination)**.** *For every normal modal logic Σ: $\mathfrak{M}^{\Sigma} \models \varphi$ if and only if $\Sigma \vdash \varphi$.*

Proof. If $\mathfrak{M}^{\Sigma} \models \varphi$, then for every complete Σ-consistent w, we have $\mathfrak{M}^{\Sigma}, w \models \varphi$. Hence, by the Truth Lemma, $\varphi \in w$ for every complete Σ-consistent w, whence by **??** (with $\Gamma = \emptyset$), $\Sigma \vdash \varphi$. Conversely, if $\Sigma \vdash \varphi$ then by **????**, every complete Σ-consistent w contains φ, and hence by the Truth Lemma $\mathfrak{M}^{\Sigma}, w \models \varphi$ for every w, i.e., $\mathfrak{M}^{\Sigma} \models \varphi$. □

mod:com:cmk:
cor:Kcomplete

Corollary com.2. *The basic modal logic* **K** *is complete with respect to the class of all models, i.e., if $\models \varphi$ then $\mathbf{K} \vdash \varphi$.*

Proof. Contrapositively, if $\mathbf{K} \nvdash \varphi$ then by Determination $\mathfrak{M}^{\mathbf{K}} \not\models \varphi$ and hence φ is not valid. □

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Bibliography