com.1 Complete Consistent Sets

mod:com:ccs:
sec

Definition com.1. A set Γ is *complete Σ-consistent* if and only if it is Σ-consistent and for every φ, either $\varphi \in \Gamma$ or $\neg\varphi \in \Gamma$.

mod:com:ccs:
prop:completeconsproperties

Proposition com.2. *Suppose Γ is complete Σ-consistent. Then:*

1. *Γ is deductively closed in Σ.*

mod:com:ccs:
prop:completeconsproperties-b

2. *$\Sigma \subseteq \Gamma$.*

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prop:completeconsproperties-c

3. *$\neg\varphi \in \Gamma$ if and only if $\varphi \notin \Gamma$.*

mod:com:ccs:
prop:completeconsproperties-d

4. *$\varphi \to \psi \in \Gamma$ if and only if $\varphi \in \Gamma$ implies $\psi \in \Gamma$.*

Proof.

1. If $\Gamma \vdash_\Sigma \varphi$ but $\varphi \notin \Gamma$ then by maximality $\neg\varphi \in \Gamma$, and Γ is inconsistent.

2. If $\varphi \in \Sigma$ then $\Gamma \vdash_\Sigma \varphi$, and $\varphi \in \Gamma$ by deductive closure.

3. If $\neg\varphi \in \Gamma$, then by consistency $\varphi \notin \Gamma$; and if $\varphi \notin \Gamma$ then by maximality $\varphi \in \Gamma$.

4. Suppose $\varphi \to \psi \in \Gamma$ and $\varphi \in \Gamma$; then $\Gamma \vdash_\Sigma \psi$, whence $\psi \in \Gamma$ by deductive closure. Conversely, if $\varphi \to \psi \notin \Gamma$ then by maximality $\neg(\varphi \to \psi) \in \Gamma$, so by Rule T, deductive closure, and consistency both $\varphi \in \Gamma$ and $\psi \notin \Gamma$.

□

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Bibliography