

prf.1 Properties of Derivability

nml:prf:prp:
sec
nml:prf:prp:
prop:derivabilityfacts

Proposition prf.1. *Let Σ be a modal system and Γ a set of modal formulas. The following properties hold:*

1. Monotonicity: *If $\Gamma \vdash_{\Sigma} \varphi$ and $\Gamma \subseteq \Delta$ then $\Delta \vdash_{\Sigma} \varphi$;*
nml:prf:prp:
prop:derivabilityfacts-monotonicity
2. Reflexivity: *If $\varphi \in \Gamma$ then $\Gamma \vdash_{\Sigma} \varphi$;*
nml:prf:prp:
prop:derivabilityfacts-reflexivity
3. Cut: *If $\Gamma \vdash_{\Sigma} \varphi$ and $\Delta \cup \{\varphi\} \vdash_{\Sigma} \psi$ then $\Gamma \cup \Delta \vdash_{\Sigma} \psi$;*
nml:prf:prp:
prop:derivabilityfacts-cut
4. Deduction theorem: *$\Gamma \cup \{\psi\} \vdash_{\Sigma} \varphi$ if and only if $\Gamma \vdash_{\Sigma} \psi \rightarrow \varphi$;*
nml:prf:prp:
prop:derivabilityfacts-deduction
5. *$\Gamma \vdash_{\Sigma} \varphi_1$ and ... and $\Gamma \vdash_{\Sigma} \varphi_n$ and $\varphi_1 \rightarrow (\varphi_2 \rightarrow \dots (\varphi_n \rightarrow \psi) \dots)$ is a tautological instance, then $\Gamma \vdash_{\Sigma} \psi$.*
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prop:derivabilityfacts-ruleT

The proof is an easy exercise. Part (5) of **Proposition prf.1** gives us that, for instance, if $\Gamma \vdash_{\Sigma} \varphi \vee \psi$ and $\Gamma \vdash_{\Sigma} \neg\varphi$, then $\Gamma \vdash_{\Sigma} \psi$. Also, in what follows, we write $\Gamma, \varphi \vdash_{\Sigma} \psi$ instead of $\Gamma \cup \{\varphi\} \vdash_{\Sigma} \psi$.

Definition prf.2. A set Γ is *deductively closed* relatively to a system Σ if and only if $\Gamma \vdash_{\Sigma} \varphi$ implies $\varphi \in \Gamma$.

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Bibliography