Prop. 1 Properties of Derivability

Proposition prf.1. Let \( \Sigma \) be a modal system and \( \Gamma \) a set of modal formulas. The following properties hold:

1. Monotony: If \( \Gamma \vdash \Sigma \varphi \) and \( \Gamma \subseteq \Delta \) then \( \Delta \vdash \Sigma \varphi \);
2. Reflexivity: If \( \varphi \in \Gamma \) then \( \Gamma \vdash \Sigma \varphi \);
3. Cut: If \( \Gamma \vdash \Sigma \varphi \) and \( \Delta \cup \{ \varphi \} \vdash \Sigma \psi \) then \( \Gamma \cup \Delta \vdash \Sigma \psi \);
4. Deduction theorem: \( \Gamma \cup \{ \psi \} \vdash \Sigma \varphi \) if and only if \( \Gamma \vdash \Sigma \varphi \rightarrow \psi \);
5. Rule T: If \( \Gamma \vdash \Sigma \varphi_1 \) and \ldots and \( \Gamma \vdash \Sigma \varphi_n \) and \( \varphi_1 \rightarrow (\varphi_2 \rightarrow \cdots (\varphi_n \rightarrow \psi) \cdots) \) is a tautological instance, then \( \Gamma \vdash \Sigma \psi \).

The proof is an easy exercise. Part (5) of Proposition prf.1 gives us that, for instance, if \( \Gamma \vdash \Sigma \varphi \lor \psi \) and \( \Gamma \vdash \Sigma \neg \varphi \), then \( \Gamma \vdash \Sigma \psi \). Also, in what follows, we write \( \Gamma, \varphi \vdash \Sigma \psi \) instead of \( \Gamma \cup \{ \varphi \} \vdash \Sigma \psi \).

Definition prf.2. A set \( \Gamma \) is deductively closed relatively to a system \( \Sigma \) if and only if \( \Gamma \vdash \Sigma \varphi \) implies \( \varphi \in \Gamma \).

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Bibliography