

prf.1 Proofs in Modal Systems

mod:prf:prs:
sec

We now come to proofs in systems of modal logic other than **K**.

mod:prf:prs:
prop:S5facts

Proposition prf.1. *The following provability results obtain:*

1. **KT5** \vdash B;
2. **KT5** \vdash 4;
3. **KDB4** \vdash T;
4. **KB4** \vdash 5;
5. **KB5** \vdash 4;
6. **KT** \vdash D.

mod:prf:prs:
prop:S5facts-KT-D

Proof. We exhibit proofs for each.

1. **KT5** \vdash B:

1. $\Diamond\varphi \rightarrow \Box\Diamond\varphi$ 5
2. $\varphi \rightarrow \Diamond\varphi$ T_{\Diamond}
3. $\varphi \rightarrow \Box\Diamond\varphi$ PL.

2. **KT5** \vdash 4:

1. $\Diamond\Box\varphi \rightarrow \Box\Diamond\Box\varphi$ 5 with $\Box\varphi$ for φ
2. $\Box\varphi \rightarrow \Diamond\Box\varphi$ T_{\Diamond} with $\Box\varphi$ for φ
3. $\Box\varphi \rightarrow \Box\Diamond\Box\varphi$ PL, 1, 2
4. $\Diamond\Box\varphi \rightarrow \Box\varphi$ 5_{\Diamond}
5. $\Box\Diamond\Box\varphi \rightarrow \Box\Box\varphi$ RK, 4
6. $\Box\varphi \rightarrow \Box\Box\varphi$ PL, 3, 5.

3. **KDB4** \vdash T:

1. $\Diamond\Box\varphi \rightarrow \varphi$ B_{\Diamond}
2. $\Box\Box\varphi \rightarrow \Diamond\Box\varphi$ D with $\Box\varphi$ for φ
3. $\Box\Box\varphi \rightarrow \varphi$ PL1, 2
4. $\Box\varphi \rightarrow \Box\Box\varphi$ 4
5. $\Box\varphi \rightarrow \varphi$ PL, 1, 4.

4. **KB4** \vdash 5:

1. $\Diamond\varphi \rightarrow \Box\Diamond\Diamond\varphi$ B with $\Diamond\varphi$ for φ
2. $\Diamond\Diamond\varphi \rightarrow \Diamond\varphi$ 4_{\Diamond}
3. $\Box\Diamond\Diamond\varphi \rightarrow \Box\Diamond\varphi$ RK, 2
4. $\Diamond\varphi \rightarrow \Box\Diamond\varphi$ PL, 1, 3.

5. **KB5** ⊢ 4:

1. $\Box\varphi \rightarrow \Box\Diamond\Box\varphi$ B with $\Box\varphi$ for φ
2. $\Diamond\Box\varphi \rightarrow \Box\varphi$ 5_\Diamond
3. $\Box\Diamond\Box\varphi \rightarrow \Box\Box\varphi$ RK, 2
4. $\Box\varphi \rightarrow \Box\Box\varphi$ PL, 1, 3.

6. **KT** ⊢ D:

1. $\Box\varphi \rightarrow \varphi$ T
2. $\varphi \rightarrow \Diamond\varphi$ T_\Diamond
3. $\Box\varphi \rightarrow \Diamond\varphi$ PL, 1, 2

□

Proposition prf.2. $\mathbf{KTB4} = \mathbf{KT5} = \mathbf{KDB4} = \mathbf{KDB5}$.

*mod:prf:prs:
prop:S5*

Problem prf.1. Prove [Proposition prf.2](#).

Definition prf.3. Following tradition, we define **S4** to be the system **KT4**, and **S5** the system **KTB4**.

[Proposition prf.2](#) shows that the classical system **S5** has several equivalent axiomatizations (see ??).

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Bibliography