prf.1 Normal Modal Logics

mod:prf:nor:sec

Definition prf.1. A modal logic Σ is *normal* if it is closed under the rule RK:

$$\frac{\varphi_1 \to (\varphi_2 \to \cdots (\varphi_{n-1} \to \varphi_n) \cdots)}{\Box\varphi_1 \to (\Box\varphi_2 \to \cdots (\Box\varphi_{n-1} \to \Box\varphi_n) \cdots)} \text{ RK}$$

Observe that while tautological implication is "fine-grained" enough to preserve *truth at a world*, the rule RK only preserves *truth in a model* (and hence also validity in a frame or in a class of frames).

Proposition prf.2. *Every normal modal logic Σ is closed under the rule of* Necessitation*:*

$$\frac{\varphi}{\Box\varphi} \text{ NEC}$$

Proof. NEC is just the special case of RK when $n = 1$. □

Proposition prf.3. *Every normal modal logic Σ contains every instance of* K*.*

Proof. In fact, K follows from rule RK: $(\varphi \to \psi) \to (\varphi \to \psi)$ is in Σ since it is a tautological instance; one application of RK gives that $\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$ is in Σ as well. □

mod:prf:nor:prop:notDiamondBot

Proposition prf.4. *Every normal modal logic Σ contains $\neg\Diamond\bot$.*

Problem prf.1. Prove Proposition prf.4.

Photo Credits

Bibliography