

## prf.1 More Proofs in K

mod:prf:mpr:  
sec Let's see some more examples of **derivability** in **K**, now using the simplified method introduced in ??.

**Proposition prf.1.**  $\mathbf{K} \vdash \Box(\varphi \rightarrow \psi) \rightarrow (\Diamond\varphi \rightarrow \Diamond\psi)$

*Proof.*

1.  $\mathbf{K} \vdash (\varphi \rightarrow \psi) \rightarrow (\neg\psi \rightarrow \neg\varphi)$  PL
2.  $\mathbf{K} \vdash \Box(\varphi \rightarrow \psi) \rightarrow (\Box\neg\psi \rightarrow \Box\neg\varphi)$  RK, 1
3.  $\mathbf{K} \vdash (\Box\neg\psi \rightarrow \Box\neg\varphi) \rightarrow (\neg\Box\neg\varphi \rightarrow \neg\Box\neg\psi)$  TAUT
4.  $\mathbf{K} \vdash (\Box\neg\psi \rightarrow \Box\neg\varphi) \rightarrow (\neg\Box\neg\varphi \rightarrow \neg\Box\neg\psi)$  PL, 2, 3
5.  $\mathbf{K} \vdash \Box(\varphi \rightarrow \psi) \rightarrow (\Diamond\varphi \rightarrow \Diamond\psi)$  re-writing  $\Diamond$  for  $\neg\Box\neg$ .

□

**Proposition prf.2.**  $\mathbf{K} \vdash \Box\varphi \rightarrow (\Diamond(\varphi \rightarrow \psi) \rightarrow \Diamond\psi)$

*Proof.*

1.  $\mathbf{K} \vdash \varphi \rightarrow (\neg\psi \rightarrow \neg(\varphi \rightarrow \psi))$  TAUT
2.  $\mathbf{K} \vdash \Box\varphi \rightarrow (\Box\neg\psi \rightarrow \Box\neg(\varphi \rightarrow \psi))$  RK, 1
3.  $\mathbf{K} \vdash \Box\varphi \rightarrow (\neg\Box\neg(\varphi \rightarrow \psi) \rightarrow \neg\Box\neg\psi)$  PL, 2
4.  $\mathbf{K} \vdash \Box\varphi \rightarrow (\Diamond(\varphi \rightarrow \psi) \rightarrow \Diamond\psi)$  re-writing  $\Diamond$  for  $\neg\Box\neg$ .

□

**Proposition prf.3.**  $\mathbf{K} \vdash (\Diamond\varphi \vee \Diamond\psi) \rightarrow \Diamond(\varphi \vee \psi)$

*Proof.*

1.  $\mathbf{K} \vdash \neg(\varphi \vee \psi) \rightarrow \neg\varphi$  TAUT
2.  $\mathbf{K} \vdash \Box\neg(\varphi \vee \psi) \rightarrow \Box\neg\varphi$  RK, 1
3.  $\mathbf{K} \vdash \neg\Box\neg\varphi \rightarrow \neg\Box\neg(\varphi \vee \psi)$  PL, 2
4.  $\mathbf{K} \vdash \Diamond\varphi \rightarrow \Diamond(\varphi \vee \psi)$  re-writing
5.  $\mathbf{K} \vdash \Diamond\psi \rightarrow \Diamond(\varphi \vee \psi)$  similarly
6.  $\mathbf{K} \vdash (\Diamond\varphi \vee \Diamond\psi) \rightarrow \Diamond(\varphi \vee \psi)$  PL, 4, 5.

□

**Proposition prf.4.**  $\mathbf{K} \vdash \Diamond(\varphi \vee \psi) \rightarrow (\Diamond\varphi \vee \Diamond\psi)$

*Proof.*

1.  $\mathbf{K} \vdash \neg\varphi \rightarrow (\neg\psi \rightarrow \neg(\varphi \vee \psi))$  TAUT
2.  $\mathbf{K} \vdash \Box\neg\varphi \rightarrow (\Box\neg\psi \rightarrow \Box\neg(\varphi \vee \psi))$  RK
3.  $\mathbf{K} \vdash \Box\neg\varphi \rightarrow (\neg\Box\neg(\varphi \vee \psi) \rightarrow \neg\Box\neg\psi)$  PL, 2
4.  $\mathbf{K} \vdash \neg\Box\neg(\varphi \vee \psi) \rightarrow (\Box\neg\varphi \rightarrow \neg\Box\neg\psi)$  PL, 3
5.  $\mathbf{K} \vdash \neg\Box\neg(\varphi \vee \psi) \rightarrow (\neg\neg\Box\neg\psi \rightarrow \neg\Box\neg\varphi)$  PL, 4
6.  $\mathbf{K} \vdash \Diamond(\varphi \vee \psi) \rightarrow (\neg\Diamond\psi \rightarrow \Diamond\varphi)$  re-writing  $\Diamond$  for  $\neg\Box\neg$
7.  $\mathbf{K} \vdash \Diamond(\varphi \vee \psi) \rightarrow (\Diamond\psi \vee \Diamond\varphi)$  PL, 6.

□

**Problem prf.1.** Show that the following **derivability** claims hold:

1.  $\mathbf{K} \vdash \Diamond \neg \perp \rightarrow (\Box \varphi \rightarrow \Diamond \varphi)$ ;
2.  $\mathbf{K} \vdash \Box(\varphi \vee \psi) \rightarrow (\Diamond \varphi \vee \Box \psi)$ ;
3.  $\mathbf{K} \vdash (\Diamond \varphi \rightarrow \Box \psi) \rightarrow \Box(\varphi \rightarrow \psi)$ .

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