

## prf.1 Modal Logics

mod:prf:log:  
sec

**Definition prf.1.** A *modal logic* is a set  $\Sigma$  of modal **formulas** which is closed under *tautological implication* in the following sense: if  $\varphi_1, \dots, \varphi_n \in \Sigma$  and  $\varphi_1 \rightarrow (\varphi_2 \rightarrow \dots (\varphi_n \rightarrow \varphi) \dots)$  is a tautological instance, then  $\varphi \in \Sigma$ .

**Proposition prf.2.** *Every modal logic is closed under the rule of Modus Ponens:*

$$\frac{\varphi \rightarrow \psi \quad \varphi}{\psi} \text{ MP}$$

*Proof.*  $(\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \psi)$  is tautological instance, hence if  $\varphi \rightarrow \psi$  and  $\varphi$  are in  $\Sigma$ , so is  $\psi$ .  $\square$

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## Bibliography