prf.1 Logics Defined by Proofs

mod:prf:prf:sec

Definition prf.1. Given a modal system $\mathbf{K}\varphi_1 \dots \varphi_n$ and a formula ψ we say that ψ is *derivable* in $\mathbf{K}\varphi_1 \dots \varphi_n$, written $\mathbf{K}\varphi_1 \dots \varphi_n \vdash \psi$, if and only if there are formulas $\chi_1, \dots, \chi_k$ such that $\chi_k = \psi$ and each χ_i is either a tautological instance, or an instance of the schemas K, $\varphi_1, \dots, \varphi_n$, or it follows from previous formulas by means of the rules MP or NEC.

The following proposition allows us to show that $\psi \in \Sigma$ by exhibiting a Σ-proof of ψ.

Proposition prf.2. $\mathbf{K}\varphi_1 \dots \varphi_n = \{\psi : \mathbf{K}\varphi_1 \dots \varphi_n \vdash \psi\}$.

Proof. We use induction on the length of proofs to show that $\{\psi : \mathbf{K}\varphi_1 \dots \varphi_n \vdash \psi\} \subseteq \mathbf{K}\varphi_1 \dots \varphi_n$. The converse inclusion follows by showing that $\{\psi : \mathbf{K}\varphi_1 \dots \varphi_n \vdash \varphi\}$ is a normal modal logic containing all the instances of the schemas $\varphi_1, \dots, \varphi_n$, and the observation that $\mathbf{K}\varphi_1 \dots \varphi_n$ is, by definition, the smallest such logic. □

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Bibliography