

## prf.1 Dual Formulas

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mod:prf:dua: **Definition prf.1.** Each of the formulas T, B, 4, and 5 has a *dual*, denoted  
def:duals by a subscripted diamond, as follows:

$$\begin{aligned} p &\rightarrow \diamond p && (T_{\diamond}) \\ \diamond \Box p &\rightarrow p && (B_{\diamond}) \\ \diamond \diamond p &\rightarrow \diamond p && (4_{\diamond}) \\ \diamond \Box p &\rightarrow \Box p && (5_{\diamond}) \end{aligned}$$

Each of the above dual formulas is obtained from the corresponding formula by substituting  $\neg p$  for  $p$ , contraposing, replacing  $\neg \Box \neg$  by  $\diamond$ , and replacing  $\neg \diamond \neg$  by  $\Box$ . D, i.e.,  $\Box \varphi \rightarrow \diamond \varphi$  is its own dual in that sense.

**Problem prf.1.** Show that for each formula  $\varphi$  in Definition prf.1:  $\mathbf{K} \vdash \varphi \leftrightarrow \varphi_{\diamond}$ .

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## Bibliography