

prf.1 Dual Formulas

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def:duals

Definition prf.1. Each of the formulas T, B, 4, and 5 has a *dual*, denoted by a subscripted diamond, as follows:

$$\begin{array}{ll} p \rightarrow \Diamond p & (T_{\Diamond}) \\ \Diamond \Box p \rightarrow p & (B_{\Diamond}) \\ \Diamond \Diamond p \rightarrow \Diamond p & (4_{\Diamond}) \\ \Diamond \Box p \rightarrow \Box p & (5_{\Diamond}) \end{array}$$

Each of the above dual formulas is obtained from the corresponding formula by substituting $\neg p$ for p , contraposing, replacing $\neg \Box \neg$ by \Diamond , and replacing $\neg \Diamond \neg$ by \Box . D, i.e., $\Box \varphi \rightarrow \Diamond \varphi$ is its own dual in that sense.

Problem prf.1. Show that for each formula φ in Definition prf.1: $\mathbf{K} \vdash \varphi \leftrightarrow \varphi_{\Diamond}$.

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Bibliography