prf.1 Dual Formulas

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nml:prf:dua:def:duals **Definition prf.1.** Each of the formulas T, B, 4, and 5 has a *dual*, denoted by a subscripted diamond, as follows:

$$p \to \Diamond p \tag{$\mathrm{T}_\Diamond$}$$

$$\Diamond\Box p \to p \tag{$\mathrm{B}_\Diamond$}$$

$$\Diamond\Diamond p \to \Diamond p \tag{$4_\Diamond$}$$

$$\Diamond\Box p \to \Box p \tag{$5_\Diamond$}$$

Each of the above dual formulas is obtained from the corresponding formula by substituting $\neg p$ for p, contraposing, replacing $\neg\Box\neg$ by $\Diamond$, and replacing $\neg\Diamond\neg$ by $\Box$. D, i.e., $\Box\varphi \to \Diamond\varphi$ is its own dual in that sense.

Problem prf.1. Show that for each formula φ in Definition prf.1: $\mathbf{K} \vdash \varphi \leftrightarrow \varphi_\Diamond$.

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Bibliography