

## prf.1 Dual Schemas

mod:prf:dua:  
sec

mod:prf:dua: **Definition prf.1.** Each of the schemas T, B, 4, and 5 has a *dual*, denoted by  
def:duals a subscripted diamond, as follows:

$$\begin{aligned} T_{\diamond} &: \quad \varphi \rightarrow \diamond\varphi \\ B_{\diamond} &: \quad \diamond\Box\varphi \rightarrow \varphi \\ 4_{\diamond} &: \quad \diamond\diamond\varphi \rightarrow \diamond\varphi \\ 5_{\diamond} &: \quad \diamond\Box\varphi \rightarrow \Box\varphi \end{aligned}$$

Each of the dual above schemas is obtained from the corresponding schema by replacing  $\neg\varphi$  for  $\varphi$ , contraposing, and re-writing. Schema D is its own dual (modulo the replacement of  $\neg\diamond\neg$  by  $\Box$ ).

**Problem prf.1.** Show that for each schema  $\varphi$  in [Definition prf.1](#):  $\mathbf{K} \vdash \varphi \leftrightarrow \varphi_{\diamond}$ .

## Photo Credits

## Bibliography