

prf.1 Dual Formulas

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mod:prf:dua: **Definition prf.1.** Each of the formulas T, B, 4, and 5 has a *dual*, denoted
def:duals by a subscripted diamond, as follows:

$$\begin{aligned} p &\rightarrow \diamond p && (\text{T}_\diamond) \\ \diamond \Box p &\rightarrow p && (\text{B}_\diamond) \\ \diamond \diamond p &\rightarrow \diamond p && (4_\diamond) \\ \diamond \Box p &\rightarrow \Box p && (5_\diamond) \end{aligned}$$

Each of the above dual formulas is obtained from the corresponding formula by substituting $\neg p$ for p , contraposing, replacing $\neg \Box \neg$ by \diamond , and replacing $\neg \diamond \neg$ by \Box . D, i.e., $\Box \varphi \rightarrow \diamond \varphi$ is its own dual in that sense.

Problem prf.1. Show that for each formula φ in Definition prf.1: $\mathbf{K} \vdash \varphi \leftrightarrow \varphi_\diamond$.

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Bibliography