

bas.1 Substructures

mod:bas:sub:
sec The **domain** of a **structure** \mathfrak{M} may be a subset of another \mathfrak{M}' . But we should obviously only consider \mathfrak{M} a “part” of \mathfrak{M}' if not only $|\mathfrak{M}| \subseteq |\mathfrak{M}'|$, but \mathfrak{M} and \mathfrak{M}' “agree” in how they interpret the symbols of the language at least on the shared part $|\mathfrak{M}|$.

mod:bas:sub:
defn:substructure **Definition bas.1.** Given **structures** \mathfrak{M} and \mathfrak{M}' for the same language \mathcal{L} , we say that \mathfrak{M} is a *substructure* of \mathfrak{M}' , and \mathfrak{M}' an *extension* of \mathfrak{M} , written $\mathfrak{M} \subseteq \mathfrak{M}'$, iff

1. $|\mathfrak{M}| \subseteq |\mathfrak{M}'|$,
2. For each constant $c \in \mathcal{L}$, $c^{\mathfrak{M}} = c^{\mathfrak{M}'}$;
3. For each n -place **function symbol** $f \in \mathcal{L}$ $f^{\mathfrak{M}}(a_1, \dots, a_n) = f^{\mathfrak{M}'}(a_1, \dots, a_n)$ for all $a_1, \dots, a_n \in |\mathfrak{M}|$.
4. For each n -place **predicate symbol** $R \in \mathcal{L}$, $\langle a_1, \dots, a_n \rangle \in R^{\mathfrak{M}}$ iff $\langle a_1, \dots, a_n \rangle \in R^{\mathfrak{M}'}$ for all $a_1, \dots, a_n \in |\mathfrak{M}|$.

mod:bas:sub:
rem:substructure *Remark 1.* If the language contains no constant or **function symbols**, then any $N \subseteq |\mathfrak{M}|$ determines a **substructure** \mathfrak{N} of \mathfrak{M} with **domain** $|\mathfrak{N}| = N$ by putting $R^{\mathfrak{N}} = R^{\mathfrak{M}} \cap N^n$.

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Bibliography