

## bas.1 Substructures

mod:bas:sub:  
sec The **domain** of a **structure**  $\mathfrak{M}$  may be a subset of another  $\mathfrak{M}'$ . But we should obviously only consider  $\mathfrak{M}$  a “part” of  $\mathfrak{M}'$  if not only  $|\mathfrak{M}| \subseteq |\mathfrak{M}'|$ , but  $\mathfrak{M}$  and  $\mathfrak{M}'$  “agree” in how they interpret the symbols of the language at least on the shared part  $|\mathfrak{M}|$ .

mod:bas:sub:  
defn:substructure **Definition bas.1.** Given **structures**  $\mathfrak{M}$  and  $\mathfrak{M}'$  for the same language  $\mathcal{L}$ , we say that  $\mathfrak{M}$  is a *substructure* of  $\mathfrak{M}'$ , and  $\mathfrak{M}'$  an *extension* of  $\mathfrak{M}$ , written  $\mathfrak{M} \subseteq \mathfrak{M}'$ , iff

1.  $|\mathfrak{M}| \subseteq |\mathfrak{M}'|$ ,
2. For each constant  $c \in \mathcal{L}$ ,  $c^{\mathfrak{M}} = c^{\mathfrak{M}'}$ ;
3. For each  $n$ -place **predicate symbol**  $f \in \mathcal{L}$   $f^{\mathfrak{M}}(a_1, \dots, a_n) = f^{\mathfrak{M}'}(a_1, \dots, a_n)$  for all  $a_1, \dots, a_n \in |\mathfrak{M}|$ .
4. For each  $n$ -place **predicate symbol**  $R \in \mathcal{L}$ ,  $\langle a_1, \dots, a_n \rangle \in R^{\mathfrak{M}}$  iff  $\langle a_1, \dots, a_n \rangle \in R^{\mathfrak{M}'}$  for all  $a_1, \dots, a_n \in |\mathfrak{M}|$ .

mod:bas:sub:  
rem:substructure *Remark 1.* If the language contains no constant or **function symbols**, then any  $N \subseteq |\mathfrak{M}|$  determines a **substructure**  $\mathfrak{N}$  of  $\mathfrak{M}$  with **domain**  $|\mathfrak{N}| = N$  by putting  $R^{\mathfrak{N}} = R^{\mathfrak{M}} \cap N^n$ .

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## Bibliography