bas.1 Substructures

The domain of a structure $M$ may be a subset of another $M'$. But we should obviously only consider $M$ a “part” of $M'$ if not only $|M| \subseteq |M'|$, but $M$ and $M'$ “agree” in how they interpret the symbols of the language at least on the shared part $|M|$.

**Definition bas.1.** Given structures $M$ and $M'$ for the same language $L$, we say that $M$ is a substructure of $M'$, and $M'$ an extension of $M$, written $M \subseteq M'$, iff

1. $|M| \subseteq |M'|$,
2. For each constant $c \in L$, $c^M = c^{M'}$;
3. For each $n$-place predicate symbol $f \in L$, $f^M(a_1, \ldots, a_n) = f^{M'}(a_1, \ldots, a_n)$ for all $a_1, \ldots, a_n \in |M|$.
4. For each $n$-place predicate symbol $R \in L$, $\langle a_1, \ldots, a_n \rangle \in R^M$ iff $\langle a_1, \ldots, a_n \rangle \in R^{M'}$ for all $a_1, \ldots, a_n \in |M|$.

**Remark 1.** If the language contains no constant or function symbols, then any $N \subseteq |M|$ determines a substructure $\mathfrak{N}$ of $M$ with domain $|\mathfrak{N}| = N$ by putting $R^\mathfrak{N} = R^M \cap N^n$.

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Bibliography