**bas.1 Substructures**

The domain of a structure $\mathcal{M}$ may be a subset of another $\mathcal{M}'$. But we should obviously only consider $\mathcal{M}$ a “part” of $\mathcal{M}'$ if not only $|\mathcal{M}| \subseteq |\mathcal{M}'|$, but $\mathcal{M}$ and $\mathcal{M}'$ “agree” in how they interpret the symbols of the language at least on the shared part $|\mathcal{M}|$.

**Definition bas.1.** Given structures $\mathcal{M}$ and $\mathcal{M}'$ for the same language $\mathcal{L}$, we say that $\mathcal{M}$ is a *substructure* of $\mathcal{M}'$, and $\mathcal{M}'$ an *extension* of $\mathcal{M}$, written $\mathcal{M} \subseteq \mathcal{M}'$, iff

1. $|\mathcal{M}| \subseteq |\mathcal{M}'|$, 
2. For each constant $c \in \mathcal{L}$, $c^\mathcal{M} = c^\mathcal{M}'$; 
3. For each $n$-place function symbol $f \in \mathcal{L}$, $f^\mathcal{M}(a_1, \ldots, a_n) = f^\mathcal{M}'(a_1, \ldots, a_n)$ for all $a_1, \ldots, a_n \in |\mathcal{M}|$. 
4. For each $n$-place predicate symbol $R \in \mathcal{L}$, $\langle a_1, \ldots, a_n \rangle \in R^\mathcal{M}$ iff $\langle a_1, \ldots, a_n \rangle \in R^\mathcal{M}'$ for all $a_1, \ldots, a_n \in |\mathcal{M}|$.

**Remark 1.** If the language contains no constant or function symbols, then any $N \subseteq |\mathcal{M}|$ determines a substructure $\mathcal{N}$ of $\mathcal{M}$ with domain $|\mathcal{N}| = N$ by putting $R^\mathcal{N} = R^\mathcal{M} \cap N^n$.

**Photo Credits**

**Bibliography**