

## bas.1 Overspill

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**Theorem bas.1.** *If a set  $\Gamma$  of sentences has arbitrarily large finite models, then it has an infinite model.*

*Proof.* Expand the language of  $\Gamma$  by adding countably many new constants  $c_0, c_1, \dots$  and consider the set  $\Gamma \cup \{c_i \neq c_j : i \neq j\}$ . To say that  $\Gamma$  has arbitrarily large finite models means that for every  $m > 0$  there is  $n \geq m$  such that  $\Gamma$  has a model of cardinality  $n$ . This implies that  $\Gamma \cup \{c_i \neq c_j : i \neq j\}$  is finitely satisfiable. By compactness,  $\Gamma \cup \{c_i \neq c_j : i \neq j\}$  has a model  $\mathfrak{M}$  whose domain must be infinite, since it satisfies all inequalities  $c_i \neq c_j$ .  $\square$

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**Proposition bas.2.** *There is no sentence  $\varphi$  of any first-order language that is true in a structure  $\mathfrak{M}$  if and only if the domain  $|\mathfrak{M}|$  of the structure is infinite.*

*Proof.* If there were such a  $\varphi$ , its negation  $\neg\varphi$  would be true in all and only the finite structures, and it would therefore have arbitrarily large finite models but it would lack an infinite model, contradicting **Theorem bas.1**.  $\square$

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## Bibliography