

bas.1 Overspill

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Theorem bas.1. *If a set Γ of sentences has arbitrarily large finite models, then it has an infinite model.*

Proof. Expand the language of Γ by adding countably many new constants c_0, c_1, \dots and consider the set $\Gamma \cup \{c_i \neq c_j : i \neq j\}$. To say that Γ has arbitrarily large finite models means that for every $m > 0$ there is $n \geq m$ such that Γ has a model of cardinality n . This implies that $\Gamma \cup \{c_i \neq c_j : i \neq j\}$ is finitely satisfiable. By compactness, $\Gamma \cup \{c_i \neq c_j : i \neq j\}$ has a model \mathfrak{M} whose domain must be infinite, since it satisfies all inequalities $c_i \neq c_j$. \square

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Proposition bas.2. *There is no sentence φ of any first-order language that is true in a *structure* \mathfrak{M} if and only if the domain $|\mathfrak{M}|$ of the *structure* is infinite.*

Proof. If there were such a φ , its negation $\neg\varphi$ would be true in all and only the finite *structures*, and it would therefore have arbitrarily large finite models but it would lack an infinite model, contradicting [Theorem bas.1](#). \square

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Bibliography