syn.1 Valuations and Satisfaction

Definition syn.1 (Valuations). Let $V$ be a set of truth values. A *valuation* for $\mathcal{L}$ into $V$ is a function $v$ assigning an element of $V$ to the propositional variables of the language, i.e., $v: \text{At}_0 \to V$.

Definition syn.2. Given a valuation $v$ into the set of truth values $V$ of a many-valued logic $\mathcal{L}$, define the evaluation function $\overline{v}: \text{Frm}(\mathcal{L}) \to V$ inductively by:

1. $\overline{v}(p_n) = v(p_n)$;
2. If $\star$ is a 0-place connective, then $\overline{v}(\star) = e_{\mathcal{L}}(\star)$;
3. If $\star$ is an $n$-place connective, then $\overline{v}(\star(\varphi_1, \ldots, \varphi_n)) = e_{\mathcal{L}}(\overline{v}(\varphi_1), \ldots, \overline{v}(\varphi_n))$.

Definition syn.3 (Satisfaction). The formula $\varphi$ is *satisfied* by a valuation $v$, $v \models_{\mathcal{L}} \varphi$, iff $\overline{v}_{\mathcal{L}}(\varphi) \in V^+$, where $V^+$ is the set of designated truth values of $\mathcal{L}$.

We write $v \not\models_{\mathcal{L}} \varphi$ to mean “not $v \models_{\mathcal{L}} \varphi$.” If $\Gamma$ is a set of formulas, $v \models_{\mathcal{L}} \Gamma$ iff $v \models_{\mathcal{L}} \varphi$ for every $\varphi \in \Gamma$.

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Bibliography