Definition syn.1 (Valuations). Let $V$ be a set of truth values. A *valuation* for $L$ into $V$ is a function $v$ assigning an element of $V$ to the propositional variables of the language, i.e., $v : At \rightarrow V$.

Definition syn.2. Given a valuation $v$ into the set of truth values $V$ of a many-valued logic $L$, define the evaluation function $\bar{v} : Frm(L) \rightarrow V$ inductively by:

1. $\bar{v}(p_n) = v(p_n)$;
2. If $*$ is a 0-place connective, then $\bar{v}(*) = e_\star L$;
3. If $*$ is an $n$-place connective, then
   $$\bar{v}(\star(\varphi_1, \ldots, \varphi_n)) = e_\star L(\bar{v}(\varphi_1), \ldots, \bar{v}(\varphi_n)).$$

Definition syn.3 (Satisfaction). The formula $\varphi$ is *satisfied* by a valuation $v$, $v \models_L \varphi$, iff $\bar{v}_L(\varphi) \in V^+$, where $V^+$ is the set of designated truth values of $L$.

We write $v \not\models_L \varphi$ to mean “not $v \models_L \varphi.” If $\Gamma$ is a set of formulas, $v \models_L \Gamma$ iff $v \models_L \varphi$ for every $\varphi \in \Gamma$.

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Bibliography