Definition syn.1 (Valuations). Let \( V \) be a set of truth values. A \textit{valuation} for \( \mathcal{L} \) into \( V \) is a function \( v \) assigning an element of \( V \) to the propositional variables of the language, i.e., \( v: \text{At}_0 \to V \).

Definition syn.2. Given a valuation \( v \) into the set of truth values \( V \) of a many-valued logic \( \mathcal{L} \), define the evaluation function \( \overline{v}: \text{Frm}(\mathcal{L}) \to V \) inductively by:

1. \( \overline{v}(p_n) = v(p_n) \);
2. If \( \star \) is a 0-place connective, then \( \overline{v}(\star) = \varepsilon_\mathcal{L} \);
3. If \( \star \) is an \( n \)-place connective, then
   \[
   \overline{v}(\star(\varphi_1, \ldots, \varphi_n)) = \varepsilon_\mathcal{L}(\overline{v}(\varphi_1), \ldots, \overline{v}(\varphi_n)).
   \]

Definition syn.3 (Satisfaction). The formula \( \varphi \) is \textit{satisfied} by a valuation \( v \), \( v \models_\mathcal{L} \varphi \), iff \( \overline{v}_\mathcal{L}(\varphi) \in V^+ \), where \( V^+ \) is the set of designated truth values of \( \mathcal{L} \).

We write \( v \not\models_\mathcal{L} \varphi \) to mean “not \( v \models_\mathcal{L} \varphi \).” If \( \Gamma \) is a set of formulas, \( v \models_\mathcal{L} \Gamma \) iff \( v \models_\mathcal{L} \varphi \) for every \( \varphi \in \Gamma \).

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Bibliography