Suppose a many-valued logic $L$ is given by a matrix. Then we can define the usual semantic notions for $L$.

**Definition syn.1.**
1. A formula $\varphi$ is **satisfiable** if for some $v$, $v \models \varphi$; it is **unsatisfiable** if for no $v$, $v \models \varphi$;
2. A formula $\varphi$ is a **tautology** if $\forall v \models \varphi$ for all valuations $v$;
3. If $\Gamma$ is a set of formulas, $\Gamma \models \varphi$ ("$\Gamma$ entails $\varphi$") if and only if $v \models \varphi$ for every valuation $v$ for which $v \models \Gamma$.
4. If $\Gamma$ is a set of formulas, $\Gamma$ is **satisfiable** if there is a valuation $v$ for which $v \models \Gamma$, and $\Gamma$ is **unsatisfiable** otherwise.

We have some of the same facts for these notions as we do for the case of classical logic:

**Proposition syn.2.**

1. $\varphi$ is a tautology if and only if $\emptyset \models \varphi$;
2. If $\Gamma$ is satisfiable then every finite subset of $\Gamma$ is also satisfiable;
3. Monotonicity: if $\Gamma \subseteq \Delta$ and $\Gamma \models \varphi$ then also $\Delta \models \varphi$;
4. Transitivity: if $\Gamma \models \varphi$ and $\Delta \cup \{\varphi\} \models \psi$ then $\Gamma \cup \Delta \models \psi$.

**Proof.** Exercise.

**Problem syn.1.** Prove Proposition syn.2

In classical logic we can connect entailment and the conditional. For instance, we have the validity of modus ponens: If $\Gamma \models \varphi$ and $\Gamma \models \varphi \rightarrow \psi$ then $\Gamma \models \psi$. Another important relationship between $\models$ and $\rightarrow$ in classical logic is the semantic deduction theorem: $\Gamma \models \varphi \rightarrow \psi$ if and only if $\Gamma \cup \{\varphi\} \models \psi$. These results do not always hold in many-valued logics. Whether they do depends on the truth function $\rightarrow$.

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Bibliography