syn.1 Semantic Notions

mvl:syn:sem:sec Suppose a many-valued logic **L** is given by a matrix. Then we can define the usual semantic notions for **L**.

Definition syn.1.

1. A formula φ is *satisfiable* if for some $\mathfrak{v}$, $\mathfrak{v} \vDash \varphi$; it is *unsatisfiable* if for no $\mathfrak{v}$, $\mathfrak{v} \vDash \varphi$;
2. A formula φ is a *tautology* if $\mathfrak{v} \vDash \varphi$ for all valuations v;
3. If Γ is a set of formulas, $\Gamma \vDash \varphi$ ("Γ entails φ") if and only if $\mathfrak{v} \vDash \varphi$ for every valuation $\mathfrak{v}$ for which $\mathfrak{v} \vDash \Gamma$.
4. If Γ is a set of formulas, Γ is *satisfiable* if there is a valuation $\mathfrak{v}$ for which $\mathfrak{v} \vDash \Gamma$, and Γ is *unsatisfiable* otherwise.

We have some of the same facts for these notions as we do for the case of classical logic:

mvl:syn:sem:prop:semanticalfacts **Proposition syn.2.**

1. *φ is a tautology if and only if $\emptyset \vDash \varphi$;*
2. *If Γ is satisfiable then every finite subset of Γ is also satisfiable;*
3. *mvl:syn:sem:def:monotonicity* *Monotonicity: if $\Gamma \subseteq \Delta$ and $\Gamma \vDash \varphi$ then also $\Delta \vDash \varphi$;*
4. *mvl:syn:sem:def:Cut* *Transitivity: if $\Gamma \vDash \varphi$ and $\Delta \cup \{\varphi\} \vDash \psi$ then $\Gamma \cup \Delta \vDash \psi$;*

Proof. Exercise. □

Problem syn.1. Prove Proposition syn.2

In classical logic we can connect entailment and the conditional. For instance, we have the validity of *modus ponens*: If $\Gamma \vDash \varphi$ and $\Gamma \vDash \varphi \rightarrow \psi$ then $\Gamma \vDash \psi$. Another important relationship between $\vDash$ and $\rightarrow$ in classical logic is the semantic deduction theorem: $\Gamma \vDash \varphi \rightarrow \psi$ if and only if $\Gamma \cup \{\varphi\} \vDash \psi$. These results *do not* always hold in many-valued logics. Whether they do depends on the truth function $\widetilde{\rightarrow}$.

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Bibliography