

syn.1 Semantic Notions

mvl:syn:sem:
sec Suppose a many-valued logic \mathbf{L} is given by a matrix. Then we can define the usual semantic notions for \mathbf{L} .

Definition syn.1. 1. A formula φ is *satisfiable* if for some \mathbf{v} , $\mathbf{v} \models \varphi$; it is *unsatisfiable* if for no \mathbf{v} , $\mathbf{v} \models \varphi$;

2. A formula φ is a *tautology* if $\mathbf{v} \models \varphi$ for all valuations v ;

3. If Γ is a set of formulas, $\Gamma \models \varphi$ (“ Γ entails φ ”) if and only if $\mathbf{v} \models \varphi$ for every valuation \mathbf{v} for which $\mathbf{v} \models \Gamma$.

4. If Γ is a set of formulas, Γ is *satisfiable* if there is a valuation \mathbf{v} for which $\mathbf{v} \models \Gamma$, and Γ is *unsatisfiable* otherwise.

We have some of the same facts for these notions as we do for the case of classical logic:

mvl:syn:sem:
prop:semanticalfacts **Proposition syn.2.**

1. φ is a tautology if and only if $\emptyset \models \varphi$;

2. If Γ is satisfiable then every finite subset of Γ is also satisfiable;

mvl:syn:sem:
def:Monotony 3. *Monotony:* if $\Gamma \subseteq \Delta$ and $\Gamma \models \varphi$ then also $\Delta \models \varphi$;

mvl:syn:sem:
def:Cut 4. *Transitivity:* if $\Gamma \models \varphi$ and $\Delta \cup \{\varphi\} \models \psi$ then $\Gamma \cup \Delta \models \psi$;

Proof. Exercise. □

Problem syn.1. Prove [Proposition syn.2](#)

In classical logic we can connect entailment and the conditional. For instance, we have the validity of *modus ponens*: If $\Gamma \models \varphi$ and $\Gamma \models \varphi \rightarrow \psi$ then $\Gamma \models \psi$. Another important relationship between \models and \rightarrow in classical logic is the semantic deduction theorem: $\Gamma \models \varphi \rightarrow \psi$ if and only if $\Gamma \cup \{\varphi\} \models \psi$. These results *do not* always hold in many-valued logics. Whether they do depends on the truth function $\widetilde{\rightarrow}$.

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Bibliography