

syn.1 Introduction

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In classical logic, we deal with **formulas** that are built from **propositional variables** using the propositional connectives \neg , \wedge , \vee , \rightarrow , and \leftrightarrow . When we define a semantics for classical logic, we do so using the two truth values \mathbb{T} and \mathbb{F} . We interpret **propositional variables** in a **valuation** \mathbf{v} , which assigns these truth values \mathbb{T} , \mathbb{F} to the **propositional variables**. Any **valuation** then determines a truth value $\bar{\mathbf{v}}(\varphi)$ for any **formula** φ , and **A formula** is satisfied in a **valuation** \mathbf{v} , $\mathbf{v} \models \varphi$, iff $\bar{\mathbf{v}}(\varphi) = \mathbb{T}$.

Many-valued logics are generalizations of classical two-valued logic by allowing more truth values than just \mathbb{T} and \mathbb{F} . So in many-valued logic, a **valuation** \mathbf{v} is a function assigning to every **propositional variable** p one of a range of possible truth values. We'll generally call the set of allowed truth values V . Classical logic is a many-valued logic where $V = \{\mathbb{T}, \mathbb{F}\}$, and the truth value $\bar{\mathbf{v}}(\varphi)$ is computed using the familiar characteristic truth tables for the connectives.

Once we add additional truth values, we have more than one natural option for how to compute $\bar{\mathbf{v}}(\varphi)$ for the connectives we read as “and,” “or,” “not,” and “if—then.” So a many-valued logic is determined not just by the set of truth values, but also by the *truth functions* we decide to use for each connective. Once these are selected for a many-valued logic \mathbf{L} , however, the truth value $\bar{\mathbf{v}}_{\mathbf{L}}(\varphi)$ is uniquely determined by the valuation, just like in classical logic. Many-valued logics, like classical logic, are *truth functional*.

With this semantic building blocks in hand, we can go on to define the analogs of the semantic concepts of tautology, entailment, and satisfiability. In classical logic, a **formula** is a tautology if its truth value $\bar{\mathbf{v}}(\varphi) = \mathbb{T}$ for any \mathbf{v} . In many-valued logic, we have to generalize this a bit as well. First of all, there is no requirement that the set of truth values V contains \mathbb{T} . For instance, some many-valued logics use numbers, such as all rational numbers between 0 and 1 as their set of truth values. In such a case, 1 usually plays the role of \mathbb{T} . In other logics, not just one but several truth values do. So, we require that every many-valued logic have a set V^+ of *designated values*. We can then say that a **formula** is satisfied in a **valuation** \mathbf{v} , $\mathbf{v} \models_{\mathbf{L}} \varphi$, iff $\bar{\mathbf{v}}_{\mathbf{L}}(\varphi) \in V^+$. A **formula** φ is a tautology of the logic, $\models_{\mathbf{L}} \varphi$, iff $\bar{\mathbf{v}}(\varphi) \in V^+$ for any \mathbf{v} . And, finally, we say that φ is entailed by a set of **formulas**, $\Gamma \models_{\mathbf{L}} \varphi$, if every **valuation** that satisfies all the **formulas** in Γ also satisfies φ .

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Bibliography