syn.1 Formulas

Definition syn.1 (Formula). The set \( \text{Frm}(\mathcal{L}) \) of formulas of a propositional language \( \mathcal{L} \) is defined inductively as follows:

1. Every propositional variable \( p_i \) is an atomic formula.
2. Every 0-place connective (propositional constant) of \( \mathcal{L} \) is an atomic formula.
3. If \( \ast \) is an \( n \)-place connective of \( \mathcal{L} \), and \( \varphi_1, \ldots, \varphi_n \) are formulas, then \( \ast(\varphi_1, \ldots, \varphi_n) \) is a formula.
4. Nothing else is a formula.

If \( \ast \) is 1-place, then \( \ast(\varphi_1) \) will often be written simply as \( \ast \varphi_1 \). If \( \ast \) is 2-place \( \ast(\varphi_1, \varphi_2) \) will often be written as \( (\varphi_1 \ast \varphi_2) \).

As usual, we will often silently leave out the outermost parentheses.

Example syn.2. In the standard language \( \mathcal{L}_0 \), \( p_1 \rightarrow (p_1 \land \neg p_2) \) is a formula. In the language of product logic, it would be written instead as \( p_1 \rightarrow (p_1 \odot \neg p_2) \). If we add the 1-place \( \triangle \) to the language, we would also have formulas such as \( \triangle(p_1 \land p_2) \rightarrow (\triangle p_1 \land \triangle p_2) \).

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Bibliography