

seq.1 Propositional Rules for Selected Logics

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sec

The inference rules for a connective in an n -sided sequent calculus only depend on the characteristic truth function for the connective. Thus, if some connective is defined by the same truth function in different logics, these n -sided sequent rules for the connective are the same in those logics.

Rules for \neg

The following rules for \neg apply to Łukasiewicz and Kleene logics, and their variants.

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi}{\neg\varphi, \Gamma \mid \Pi \mid \Delta} \neg\mathbb{F}$$

$$\frac{\Gamma \mid \varphi, \Pi \mid \Delta}{\Gamma \mid \neg\varphi, \Pi \mid \Delta} \neg\mathbb{U}$$

$$\frac{\varphi, \Gamma \mid \Pi \mid \Delta}{\Gamma \mid \Pi \mid \Delta, \neg\varphi} \neg\mathbb{T}$$

The following rules for \neg apply to Gödel logic.

$$\frac{\Gamma \mid \varphi, \Pi \mid \Delta, \varphi}{\neg\varphi, \Gamma \mid \Pi \mid \Delta} \neg\mathbf{G}\mathbb{F}$$

$$\frac{\varphi, \Gamma \mid \Pi \mid \Delta}{\Gamma \mid \Pi \mid \Delta, \neg\varphi} \neg\mathbf{G}\mathbb{T}$$

(In Gödel logic, $\neg\varphi$ can never take the value \mathbb{U} , so there is no rule for the middle position.)

Rules for \wedge

These are the rules for \wedge in Łukasiewicz, strong Kleene, and Gödel logic.

$$\frac{\varphi, \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \wedge \psi, \Gamma \mid \Pi \mid \Delta} \wedge\mathbb{F}$$

$$\frac{\Gamma \mid \varphi, \Pi \mid \varphi, \Delta \quad \Gamma \mid \psi, \Pi \mid \psi, \Delta \quad \Gamma \mid \varphi, \psi, \Pi \mid \Delta}{\Gamma \mid \varphi \wedge \psi, \Pi \mid \Delta} \wedge\mathbb{U}$$

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi \quad \Gamma \mid \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \wedge \psi} \wedge\mathbb{T}$$

Rules for \vee

These are the rules for \vee in Łukasiewicz, strong Kleene, and Gödel logic.

$$\frac{\varphi, \Gamma \mid \Pi \mid \Delta \quad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \vee \psi, \Gamma \mid \Pi \mid \Delta} \vee\mathbb{F}$$

$$\frac{\varphi, \Gamma \mid \varphi, \Pi \mid \Delta \quad \psi, \Gamma \mid \psi, \Pi \mid \Delta \quad \Gamma \mid \varphi, \psi, \Pi \mid \Delta}{\Gamma \mid \varphi \vee \psi, \Pi \mid \Delta} \vee\mathbb{U}$$

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \vee \psi} \vee\mathbb{T}$$

Rules for \rightarrow

These are the rules for \rightarrow in Łukasiewicz logic.

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi \quad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \rightarrow \psi, \Gamma \mid \Pi \mid \Delta} \rightarrow_{\mathbf{L}_3}\mathbb{F}$$

$$\frac{\Gamma \mid \varphi, \psi, \Pi \mid \Delta \quad \psi, \Gamma \mid \Pi \mid \Delta, \varphi}{\Gamma \mid \varphi \rightarrow \psi, \Pi \mid \Delta} \rightarrow_{\mathbf{L}_3}\mathbb{U}$$

$$\frac{\varphi, \Gamma \mid \psi, \Pi \mid \Delta, \psi \quad \varphi, \Gamma \mid \varphi, \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \rightarrow \psi} \rightarrow_{\mathbf{L}_3}\mathbb{T}$$

These are the rules for \rightarrow in strong Kleene logic.

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi \quad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \rightarrow \psi, \Gamma \mid \Pi \mid \Delta} \rightarrow_{\mathbf{K}_s}\mathbb{F}$$

$$\frac{\psi, \Gamma \mid \psi, \Pi \mid \Delta \quad \Gamma \mid \varphi, \psi, \Pi \mid \Delta \quad \Gamma \mid \varphi, \Pi \mid \Delta, \varphi}{\Gamma \mid \varphi \rightarrow \psi, \Pi \mid \Delta} \rightarrow_{\mathbf{K}_s}\mathbb{U}$$

$$\frac{\varphi, \Gamma \mid \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \rightarrow \psi} \rightarrow_{\mathbf{K}_s}\mathbb{T}$$

These are the rules for \rightarrow in Gödel logic.

$$\begin{array}{c}
\frac{\Gamma \mid \varphi, \Pi \mid \Delta, \varphi \quad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \rightarrow \psi, \Gamma \mid \Pi \mid \Delta} \rightarrow_{\mathbf{G}_3} \mathbb{F} \\
\frac{\Gamma \mid \psi, \Pi \mid \Delta \quad \Gamma \mid \Pi \mid \Delta, \varphi}{\Gamma \mid \varphi \rightarrow \psi, \Pi \mid \Delta} \rightarrow_{\mathbf{G}_3} \mathbb{U} \\
\frac{\varphi, \Gamma \mid \psi, \Pi \mid \Delta, \psi \quad \varphi, \Gamma \mid \varphi, \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \rightarrow \psi} \rightarrow_{\mathbf{G}_3} \mathbb{T}
\end{array}$$

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Bibliography

$$\begin{array}{c}
\frac{B \mid B \mid B}{B \mid A, B \mid B} \text{WU} \\
\frac{B \mid B, A \mid B}{B \mid A, B, A \mid B} \text{XU} \\
\frac{B \mid A, B, A \mid B}{A, B \mid A, B, A \mid B} \text{WU} \\
\frac{B, A \mid A, B, A \mid B}{B, A \mid A, B, A \mid B} \text{WF} \\
\frac{A \mid A \mid A}{A \mid A \mid B, A} \text{WT} \\
\frac{B, A \mid A \mid B, A}{B, B, A \mid A \mid B, A} \text{WF} \\
\frac{A \mid A \mid A}{A \mid A \mid B, A} \text{WT} \\
\frac{A \mid B, A \mid B, A}{A \mid A, B, A \mid B, A} \text{WU} \\
\frac{A \mid A \mid A}{A \mid A \mid A, A} \text{WT} \\
\frac{A \mid A \mid B, A, A}{A \mid A \mid A \mid B, A, A} \text{WT} \\
\frac{A \mid A, B, A \mid B, A}{A \mid A \rightarrow B, A \mid B, A} \text{WU} \\
\frac{A \mid A \rightarrow B, A \mid B, A}{A \rightarrow B, A \mid A \rightarrow B, A \mid B} \rightarrow \text{U} \\
\frac{A \rightarrow B, A \mid A \rightarrow B, A \mid B}{A \rightarrow B, A \mid A \rightarrow B, A \mid B} \rightarrow \text{F}
\end{array}$$

Figure 1: Example derivation in \mathbf{L}_3