We may wonder if for each term there is a unique way of forming it, and there is. For each lambda term there is only one way to construct and interpret it. In the following discussion, a formation is the procedure of constructing a term using the formation rules (one or several times) of $\Rightarrow$.

**Lemma syn.1.** A term starts with either a variable or a parenthesis.

**Proof.** Something counts as a term only if it is constructed according to $\Rightarrow$. If it is the result of $\Rightarrow$, it must be a variable. If it is the result of $\Rightarrow$ or $\Rightarrow$, it starts with a parenthesis.

**Lemma syn.2.** The result of an application starts with either two parentheses or a parenthesis and a variable.

**Proof.** If $M$ is the result of an application, it is of the form $(PQ)$, so it begins with a parenthesis. Since $P$ is a term, by Lemma syn.1, it begins either with a parenthesis or a variable.

**Lemma syn.3.** No proper initial part of a term is itself a term.

**Problem syn.1.** Prove Lemma syn.3 by induction on the length of terms.

**Proposition syn.4 (Unique Readability).** There is a unique formation for each term. In other words, if a term $M$ is formed by a formation, then it is the only formation that can form this term.

**Proof.** We prove this by induction on the formation of terms.

1. $M$ is of the form $x$, where $x$ is some variable. Since the results of abstractions and applications always start with parentheses, they cannot have been used to construct $M$; Thus, the formation of $M$ must be a single step of $\Rightarrow$.

2. $M$ is of the form $(\lambda x. N)$, where $x$ is some variable and $N$ is a term. It could not have been constructed according to $\Rightarrow$, because it is not a single variable. It is not the result of an application, by Lemma syn.2. Thus $M$ can only be the result of an abstraction on $N$. By inductive hypothesis we know that formation of $N$ is itself unique.

3. $M$ is of the form $(PQ)$, where $P$ and $Q$ are terms. Since it starts with a parentheses, it cannot also be constructed by $\Rightarrow$. By Lemma syn.1, $P$ cannot begin with $\lambda$, so $(PQ)$ cannot be the result of an abstraction. Now suppose there were another way of constructing $M$ by application, e.g., it is also of the form $(P'Q')$. Then $P$ is a proper initial segment of $P'$ (or vice versa), and this is impossible by Lemma syn.3. So $P$ and $Q$ are uniquely determined, and by inductive hypothesis we know that formations of $P$ and $Q$ is unique.
A more readable paraphrase of the above proposition is as follows:

**Proposition syn.5.** A term $M$ can only be one of the following forms:

1. $x$, where $x$ is a variable uniquely determined by $M$.
2. $(\lambda x. N)$, where $x$ is a variable and $N$ is another term, both of which is uniquely determined by $M$.
3. $(PQ)$, where $P$ and $Q$ are two terms uniquely determined by $M$.

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Bibliography