The De Bruijn Index syn.1

lam:syn:deb: α -Equivalence is very natural, as terms that are α -equivalent "mean the same." In fact, it is possible to give a syntax for lambda terms which does not distinguish terms that can be α -converted to each other. The best known replaces variables by their *De Bruijn index*.

> When we write $\lambda x. M$, we explicitly state that x is the parameter of the function, so that we can use x in M to refer to this parameter. In the de Bruijn index, however, parameters have no name and reference to them in the function body is denoted by a number denoting the levels of abstraction between them. For example, consider the example of λx . λy . yx: the outer abstraction is on binds the variable x; the inner abstraction binds the variable is y; the sub-term yx lies in the scope of the inner abstraction: there is no abstraction between yand its abstract λy , but one abstract between x and its abstract λx . Thus we write 0.1 for yx, and λ . λ . 01 for the entire term.

Definition syn.1. De Bruijn terms are inductively defines as follows:

- 1. n, where n is any natural number.
- 2. PQ, where P and Q are both De Bruijn terms.
- 3. λ . N, where N is a De Bruijn term.

A formalized translation from ordinary lambda terms to De Bruijn indexed terms is as follows:

Definition syn.2.

$$F_{\Gamma}(x) = \Gamma(x)$$

$$F_{\Gamma}(PQ) = F_{\Gamma}(P)F_{\Gamma}(Q)$$

$$F_{\Gamma}(\lambda x. N) = \lambda. F_{x,\Gamma}(N)$$

where Γ is a list of variables indexed from zero, and $\Gamma(x)$ denotes the position of the variable x in Γ . For example, if Γ is x, y, z, then $\Gamma(x)$ is 0 and $\Gamma(z)$ is 2.

 x, Γ denotes the list resulted from pushing x to the head of Γ ; for instance, continuing the Γ in last example, w, Γ is w, x, y, z.

Recovering a standard lambda term from a de Bruijn term is done as follows:

Definition syn.3.

$$G_{\Gamma}(n) = \Gamma[n]$$

$$G_{\Gamma}(PQ) = G_{\Gamma}(P)G_{\Gamma}(Q)$$

$$G_{\Gamma}(\lambda, N) = \lambda x. G_{x,\Gamma}(N)$$

where Γ is again a list of variables indexed from zero, and $\Gamma[n]$ denotes the variable in position n. For example, if Γ is x, y, z, then $\Gamma[1]$ is y.

The variable x in last equation is chosen to be any variable that not in Γ .

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Here we give some results without proving them:

Proposition syn.4. If $M \xrightarrow{\alpha} M'$, and Γ is any list containing FV(M), then $F_{\Gamma}(M) \equiv F_{\Gamma}(M')$.

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Bibliography