

ldf.1 Partial Recursive Functions are λ -Definable

lam:ldf:par:
sec Partial recursive functions are those obtained from the basic functions by composition, primitive recursion, and unbounded minimization. They differ from general recursive function in that the functions used in unbounded search are not required to be regular. Not requiring regularity means that functions defined by minimization may sometimes not be defined.

At first glance it might seem that the same methods used to show that the (total) general recursive functions are all λ -definable can be used to prove that all partial recursive functions are λ -definable. For instance, the composition of f with g is λ -defined by $\lambda x. F(Gx)$ if f and g are λ -defined by terms F and G , respectively. However, when the functions are partial, this is problematic. When $g(x)$ is undefined, meaning Gx has no normal form. In most cases this means that $F(Gx)$ has no normal forms either, which is what we want. But consider when F is $\lambda x. \lambda y. y$, in which case $F(Gx)$ does have a normal form $(\lambda y. y)$.

This problem is not insurmountable, and there are ways to λ -define all partial recursive functions in such a way that undefined values are represented by terms without a normal form. These ways are, however, somewhat more complicated and less intuitive than the approach we have taken for general recursive functions. We record the theorem here without proof:

Theorem ldf.1. *All partial recursive functions are λ -definable.*

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Bibliography