ldf.1 Pairs and Predecessor

**Definition ldf.1.** The pair of $M$ and $N$ (written $\langle M, N \rangle$) is defined as follows:

$$\langle M, N \rangle \equiv \lambda f. fMN.$$  \hspace{1cm}

Intuitively it is a function that accepts a function, and applies that function to the two elements of the pair. Following this idea we have this constructor, which takes two terms and returns the pair containing them:

$$\text{Pair} \equiv \lambda mn. \lambda f. fmn$$  

Given a pair, we also want to recover its elements. For this we need two access functions, which accept a pair as argument and return the first or second elements in it:

$$\text{Fst} \equiv \lambda p. p(\lambda mn. m)$$

$$\text{Snd} \equiv \lambda p. p(\lambda mn. n)$$

**Problem ldf.1.** Explain why the access functions Fst and Snd work.

Now with pairs we can $\lambda$-define the predecessor function:

$$\text{Pred} \equiv \lambda n. \text{Fst}(n(\lambda p. (\text{Snd} p, \text{Succ(Snd} p)))(0,0))$$

Remember that $\pi f x$ reduces to $f^n(x)$; in this case $f$ is a function that accepts a pair $p$ and returns a new pair containing the second component of $p$ and the successor of the second component; $x$ is the pair $\langle 0, 0 \rangle$. Thus, the result is $\langle 0, 0 \rangle$ for $n = 0$, and $\langle n - 1, \pi \rangle$ otherwise. Pred then returns the first component of the result.

Subtraction can be defined as Pred applied to $a$, $b$ times:

$$\text{Sub} \equiv \lambda ab. b\text{Pred} a.$$