

int.1 Fixed-Point Combinators

lam:int:fix:
sec Suppose you have a lambda term g , and you want another term k with the property that k is β -equivalent to gk . Define terms

$$\text{diag}(x) = xx$$

and

$$l(x) = g(\text{diag}(x))$$

using our notational conventions; in other words, l is the term $\lambda x. g(xx)$. Let k be the term ll . Then we have

$$\begin{aligned} k &= (\lambda x. g(xx))(\lambda x. g(xx)) \\ &\rightarrow g((\lambda x. g(xx))(\lambda x. g(xx))) \\ &= gk. \end{aligned}$$

If one takes

$$Y = \lambda g. ((\lambda x. g(xx))(\lambda x. g(xx)))$$

then Yg and $g(Yg)$ reduce to a common term; so $Yg \equiv_{\beta} g(Yg)$. This is known as “Curry’s combinator.” If instead one takes

$$Y = (\lambda xg. g(xgx))(\lambda xg. g(xgx))$$

then in fact Yg reduces to $g(Yg)$, which is a stronger statement. This latter version of Y is known as “Turing’s combinator.”

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Bibliography