

rep.1 Currying

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A λ -abstract $\lambda x.M$ represents a function of one argument, which is quite a limitation when we want to define function accepting multiple arguments. One way to do this would be by extending the λ -calculus to allow the formation of pairs, triples, etc., in which case, say, a three-place function $\lambda x.M$ would expect its argument to be a triple. However, it is more convenient to do this by *Currying*.

Let's consider an example. We'll pretend for a moment that we have a $+$ operation in the λ -calculus. The addition function is 2-place, i.e., it takes two arguments. But a λ -abstract only gives us functions of one argument: the syntax does not allow expressions like $\lambda(x,y).(x+y)$. However, we can consider the one-place function $f_x(y)$ given by $\lambda y.(x+y)$, which adds x to its single argument y . Actually, this is not a single function, but a family of different functions "add x ," one for each number x . Now we can define another one-place function g as $\lambda x.f_x$. Applied to argument x , $g(x)$ returns the function f_x —so its values are other functions. Now if we apply g to x , and then the result to y we get: $(g(x))y = f_x(y) = x + y$. In this way, the one-place function g can do the same job as the two-place addition function. "Currying" simply refers to this trick for turning two-place functions into one place functions (whose values are one-place functions).

Here is an example properly in the syntax of the λ -calculus. How do we represent the function $f(x,y) = x$? If we want to define a function that accepts two arguments and returns the first, we can write $\lambda x.\lambda y.x$, which literally is a function that accepts an argument x and returns the function $\lambda y.x$. The function $\lambda y.x$ accepts another argument y , but drops it, and always returns x . Let's see what happens when we apply $\lambda x.\lambda y.x$ to two arguments:

$$\begin{aligned} (\lambda x.\lambda y.x)MN &\xrightarrow{\beta} (\lambda y.M)N \\ &\xrightarrow{\beta} M \end{aligned}$$

In general, to write a function with parameters x_1, \dots, x_n defined by some term N , we can write $\lambda x_1.\lambda x_2.\dots\lambda x_n.N$. If we apply n arguments to it we get:

$$\begin{aligned} (\lambda x_1.\lambda x_2.\dots\lambda x_n.N)M_1\dots M_n &\xrightarrow{\beta} \\ &\xrightarrow{\beta} ((\lambda x_2.\dots\lambda x_n.N)[M_1/x_1])M_2\dots M_n \\ &\equiv (\lambda x_2.\dots\lambda x_n.N[M_1/x_1])M_2\dots M_n \\ &\vdots \\ &\xrightarrow{\beta} P[M_1/x_1]\dots[M_n/x_n] \end{aligned}$$

The last line literally means substituting M_i for x_i in the body of the function definition, which is exactly what we want when applying multiple arguments to a function.

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Bibliography