Lemma int.1. The \( \lambda \)-definable functions are closed under composition.

\begin{proof}
Suppose \( f \) is defined by composition from \( h, g_0, \ldots, g_{k-1} \). Assuming \( h, g_0, \ldots, g_{k-1} \) are \( \lambda \)-defined by \( H, G_0, \ldots, G_{k-1} \), respectively, we need to find a term \( F \) that \( \lambda \)-defines \( f \). But we can simply define \( F \) by

\[
F(x_0, \ldots, x_{l-1}) = H(G_0(x_0, \ldots, x_{l-1}), \ldots, G_{k-1}(x_0, \ldots, x_{l-1})).
\]

In other words, the language of the lambda calculus is well suited to represent composition.
\end{proof}

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Bibliography