int.1 The $\lambda$-Definable Functions are Closed under Composition

Lemma int.1. The $\lambda$-definable functions are closed under composition.

Proof. Suppose $f$ is defined by composition from $h, g_0, \ldots, g_{k-1}$. Assuming $h, g_0, \ldots, g_{k-1}$ are $\lambda$-defined by $H, G_0, \ldots, G_{k-1}$, respectively, we need to find a term $F$ that $\lambda$-defines $f$. But we can simply define $F$ by

$$F(x_0, \ldots, x_{l-1}) = H(G_0(x_0, \ldots, x_{l-1}), \ldots, G_{k-1}(x_0, \ldots, x_{l-1})).$$

In other words, the language of the lambda calculus is well suited to represent composition.

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Bibliography