int.1 The Church-Rosser Property

lam:int:cr: sec

lam:int:cr: thm:church-rosser

Theorem int.1. *Let M, N_1, and N_2 be terms, such that $M \twoheadrightarrow N_1$ and $M \twoheadrightarrow N_2$. Then there is a term P such that $N_1 \twoheadrightarrow P$ and $N_2 \twoheadrightarrow P$.*

Corollary int.2. *Suppose M can be reduced to normal form. Then this normal form is unique.*

Proof. If $M \twoheadrightarrow N_1$ and $M \twoheadrightarrow N_2$, by the previous theorem there is a term P such that N_1 and N_2 both reduce to P. If N_1 and N_2 are both in normal form, this can only happen if $N_1 \equiv P \equiv N_2$. □

Finally, we will say that two terms M and N are *β-equivalent*, or just *equivalent*, if they reduce to a common term; in other words, if there is some P such that $M \twoheadrightarrow P$ and $N \twoheadrightarrow P$. This is written $M \stackrel{\beta}{=} N$. Using Theorem int.1, you can check that $\stackrel{\beta}{=}$ is an equivalence relation, with the additional property that for every M and N, if $M \twoheadrightarrow N$ or $N \twoheadrightarrow M$, then $M \stackrel{\beta}{=} N$. (In fact, one can show that $\stackrel{\beta}{=}$ is the *smallest* equivalence relation having this property.)

Photo Credits

Bibliography