int.1 The Basic Primitive Recursive Functions are λ-Definable

Lemma int.1. The functions zero, succ, and $P^n_i$ are λ-definable.

Proof. zero is just $\lambda x. \lambda y. y$.

The successor function succ, is defined by $\text{Succ}(u) = \lambda x. \lambda y. x(uxy)$. You should think about why this works; for each numeral $\pi$, thought of as an iterator, and each function $f$, $\text{Succ}(\pi, f)$ is a function that, on input $y$, applies $f$ $n$ times starting with $y$, and then applies it once more.

There is nothing to say about projections: $\text{Proj}^n_i(x_0, \ldots, x_{n-1}) = x_i$. In other words, by our conventions, $\text{Proj}^n_i$ is the lambda term $\lambda x_0. \ldots \lambda x_{n-1}. x_i$. \hfill \square

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Bibliography