Lemma int.1. *The functions zero, succ, and \( P^n_i \) are \( \lambda \)-definable.*

Proof. Zero is just \( \lambda x. \lambda y. y \).

The successor function succ, is defined by \( \text{Succ}(u) = \lambda x. \lambda y. x(uxy) \). You should think about why this works; for each numeral \( n \), thought of as an iterator, and each function \( f \), \( \text{Succ}(n, f) \) is a function that, on input \( y \), applies \( f \) \( n \) times starting with \( y \), and then applies it once more.

There is nothing to say about projections: \( \text{Proj}^n_i(x_0, \ldots, x_{n-1}) = x_i \). In other words, by our conventions, \( \text{Proj}^n_i \) is the lambda term \( \lambda x_0. \ldots \lambda x_{n-1}. x_i \). \( \square \)