

## int.1 The Basic Primitive Recursive Functions are $\lambda$ -Definable

lam:int:bas:  
sec

**Lemma int.1.** *The functions zero, succ, and  $P_i^n$  are  $\lambda$ -definable.*

*Proof.* zero is just  $\lambda x. \lambda y. y$ .

The successor function succ, is defined by  $\text{Succ}(u) = \lambda x. \lambda y. x(uxy)$ . You should think about why this works; for each numeral  $\bar{n}$ , thought of as an iterator, and each function  $f$ ,  $\text{Succ}(\bar{n}, f)$  is a function that, on input  $y$ , applies  $f$   $n$  times starting with  $y$ , and then applies it once more.

There is nothing to say about projections:  $\text{Proj}_i^n(x_0, \dots, x_{n-1}) = x_i$ . In other words, by our conventions,  $\text{Proj}_i^n$  is the lambda term  $\lambda x_0. \dots \lambda x_{n-1}. x_i$ .  $\square$

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### Bibliography