Lemma int.1. The functions zero, succ, and $P^n_i$ are \(\lambda\)-definable.

Proof. zero is just \(\lambda x. \lambda y. y\).

The successor function succ, is defined by \(\text{Succ}(u) = \lambda x. \lambda y. x(uxy)\). You should think about why this works; for each numeral \(n\), thought of as an iterator, and each function \(f\), \(\text{Succ}(n, f)\) is a function that, on input \(y\), applies \(f\) \(n\) times starting with \(y\), and then applies it once more.

There is nothing to say about projections: \(\text{Proj}_n^0(x_0, \ldots, x_{n-1}) = x_i\). In other words, by our conventions, \(\text{Proj}_n^i\) is the lambda term \(\lambda x_0. \ldots. \lambda x_{n-1}. x_i\).\hfill \Box

Photo Credits

Bibliography