The Church–Rosser property holds for βη-reduction (\(\beta\eta\rightarrow\)).

Lemma cr.1. If \(M \beta\eta\rightarrow M'\), then \(M \beta\eta\rightarrow M'\).

Proof. By induction on the derivation of \(M \beta\eta\rightarrow M'\). If \(M \rightarrow M'\) by \(\eta\)-conversion (i.e., ??), we use ??.

The other cases are as in ??.

Lemma cr.2. If \(M \beta\eta\rightarrow M'\), then \(M \beta\eta\rightarrow M'\).

Proof. Induction on the derivation of \(M \beta\eta\rightarrow M'\).

If the last rule is ??, then \(M = \lambda x.N x\) and \(M' = N'\) for some \(x\), \(N\), \(N'\) where \(x \notin FV(N)\) and \(N \beta\eta\rightarrow N'\). Thus we can first reduce \(\lambda x.N x\) to \(N\) by \(\eta\)-conversion, followed by the series of \(\beta\eta\rightarrow\) steps that show that \(N \beta\eta\rightarrow N'\), which holds by induction hypothesis.

Lemma cr.3. \(\beta\eta\rightarrow\) is the smallest transitive relation containing \(\beta\eta\rightarrow\).

Proof. As in ??.

Theorem cr.4. \(\beta\eta\rightarrow\) satisfies Church–Rosser property.

Proof. By ??, ?? and Lemma cr.3.

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Bibliography