tab.1 Introduction

int:tab:int:sec Tableaux are certain (downward-branching) trees of signed formulas, i.e., pairs consisting of a truth value sign ($\mathbb{T}$ or $\mathbb{F}$) and a sentence

$$\mathbb{T}\,\varphi \text{ or } \mathbb{F}\,\varphi.$$

A tableau begins with a number of *assumptions*. Each further signed formula is generated by applying one of the inference rules. Some inference rules add one or more signed formulas to a tip of the tree; others add two new tips, resulting in two branches. Rules result in signed formulas where the formula is less complex than that of the signed formula to which it was applied. When a branch contains both $\mathbb{T}\,\varphi$ and $\mathbb{F}\,\varphi$, we say the branch is *closed*. If every branch in a tableau is closed, the entire tableau is closed. A closed tableau constitutes a derivation that shows that the set of signed formulas which were used to begin the tableau are unsatisfiable. This can be used to define a $\vdash$ relation: $\Gamma \vdash \varphi$ iff there is some finite set $\Gamma_0 = \{\psi_1, \dots, \psi_n\} \subseteq \Gamma$ such that there is a closed tableau for the assumptions

$$\{\mathbb{F}\,\varphi, \mathbb{T}\,\psi_1, \dots, \mathbb{T}\,\psi_n\}.$$

For intuitionistic logic, we have to both extend the notion of signed formula and adjust the rules for the connectives. In addition to a sign($\mathbb{T}$ or $\mathbb{F}$), formulas in modal tableaux also have *prefixes* σ. The prefixes are non-empty sequences of positive integers, i.e., $\sigma \in (\mathbb{Z}^+)^* \setminus \{\Lambda\}$. When we write such prefixes without the surrounding $\langle\ \rangle$, and separate the individual elements by .'s instead of ,'s. If σ is a prefix, then $\sigma.n$ is $\sigma \frown \langle n \rangle$; e.g., if $\sigma = 1.2.1$, then $\sigma.3$ is 1.2.1.3. So for instance,

$$1.2\,\mathbb{T}\,\varphi \to (\psi \to \chi)$$

is a *prefixed signed formula* (or just a *prefixed formula* for short).

Intuitively, the prefix names a world in a model that might satisfy the formulas on a branch of a tableau, and if σ names some world, then $\sigma.n$ names a world accessible from (the world named by) σ.

In intuitionistic models, the accessibility relation is reflexive and transitive. In terms of prefixes, this means that σ is accessible from σ itself, and so is any prefix that extends σ, i.e., any prefix of the form $\sigma.n_1.\cdots.n_k$. Let's introduce the notation $\sigma.*$ to indicate σ itself and any extension of it. In other words, the prefixes $\sigma.*$ are all and only the prefixes accessible from σ.

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Bibliography