sc.1  The Completeness Theorem

Theorem sc.1. If $\Gamma \models \varphi$ then $\Gamma \vdash \varphi$.

Proof. We prove the contrapositive: Suppose $\Gamma \nvDash \varphi$. Then by ??, there is a prime set $\Gamma^* \supseteq \Gamma$ such that $\Gamma^* \nvDash \varphi$. Consider the canonical model $\mathfrak{M}(\Gamma^*)$ for $\Gamma^*$ as defined in ???. For any $\psi \in \Gamma$, $\Gamma^* \vdash \psi$. Note that $\Gamma^* (\Lambda) = \Gamma^*$. By the Truth Lemma (??), we have $\mathfrak{M}(\Gamma^*), \Lambda \models \psi$ for all $\psi \in \Gamma$ and $\mathfrak{M}(\Gamma^*), \Lambda \nvDash \varphi$. This shows that $\Gamma \nvDash \varphi$. □

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Bibliography