sc.1 The Completeness Theorem

int:sc:cpl:sec

int:sc:cpl:thm:completeness **Theorem sc.1.** *If $\Gamma \vDash \varphi$ then $\Gamma \vdash \varphi$.*

Proof. We prove the contrapositive: Suppose $\Gamma \nvdash \varphi$. Then by **??**, there is a prime set $\Gamma^* \supseteq \Gamma$ such that $\Gamma^* \nvdash \varphi$. Consider the canonical model $\mathfrak{M}(\Gamma^*)$ for Γ^* as defined in **??**. For any $\psi \in \Gamma$, $\Gamma^* \vdash \psi$. Note that $\Gamma * (\Lambda) = \Gamma *$. By the Truth Lemma (**??**), we have $\mathfrak{M}(\Gamma^*), \Lambda \Vdash \psi$ for all $\psi \in \Gamma$ and $\mathfrak{M}(\Gamma^*), \Lambda \nVdash \varphi$. This shows that $\Gamma \nvDash \varphi$. □

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Bibliography