

sc.1 The Completeness Theorem

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thm:completeness

Theorem sc.1. *If $\Gamma \models \varphi$ then $\Gamma \vdash \varphi$.*

Proof. We prove the contrapositive: Suppose $\Gamma \not\vdash \varphi$. Then by ??, there is a prime set $\Gamma^* \supseteq \Gamma$ such that $\Gamma^* \not\vdash \varphi$. Consider the canonical model $\mathfrak{M}(\Gamma^*)$ for Γ^* as defined in ??. For any $\psi \in \Gamma$, $\Gamma^* \vdash \psi$. Note that $\Gamma^* \ast (\mathcal{A}) = \Gamma^*$. By the Truth Lemma (??), we have $\mathfrak{M}(\Gamma^*), \mathcal{A} \Vdash \psi$ for all $\psi \in \Gamma$ and $\mathfrak{M}(\Gamma^*), \mathcal{A} \not\vdash \varphi$. This shows that $\Gamma \not\models \varphi$. \square

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Bibliography