sc.1 The Canonical Model

The worlds in our model will be finite sequences $\sigma$ of natural numbers, i.e., $\sigma \in \mathbb{N}^*$. Note that $\mathbb{N}^*$ is inductively defined by:

1. $\Lambda \in \mathbb{N}^*$.
2. If $\sigma \in \mathbb{N}^*$ and $n \in \mathbb{N}$, then $\sigma.n \in \mathbb{N}^*$ (where $\sigma.n$ is $\sigma \bowtie \langle n \rangle$ and $\sigma \bowtie \sigma'$ is the concatenation if $\sigma$ and $\sigma'$).
3. Nothing else is in $\mathbb{N}^*$.

So we can use $\mathbb{N}^*$ to give inductive definitions.

Let $\langle \psi_1, \chi_1 \rangle$, $\langle \psi_2, \chi_s \rangle$, $\ldots$, be an enumeration of all pairs of formulas. Given a set of formulas $\Delta$, define $\Delta(\sigma)$ by induction as follows:

1. $\Delta(\Lambda) = \Delta$
2. $\Delta(\sigma.n) = $ 
   \[
   \begin{cases} 
   (\Delta(\sigma) \cup \{\psi_n\})^* & \text{if } \Delta(\sigma) \cup \{\psi_n\} \not\models \chi_n \\
   \Delta(\sigma) & \text{otherwise}
   \end{cases}
   \]

Here by $(\Delta(\sigma) \cup \{\psi_n\})^*$ we mean the prime set of formulas which exists by ?? applied to the set $\Delta(\sigma) \cup \{\psi_n\}$ and the formula $\chi_n$. Note that by this definition, if $\Delta(\sigma) \cup \{\psi_n\} \not\models \chi_n$, then $\Delta(\sigma.n) \models \psi_n$ and $\Delta(\sigma.n) \not\models \chi_n$. Note also that $\Delta(\sigma) \subseteq \Delta(\sigma.n)$ for any $n$. If $\Delta$ is prime, then $\Delta(\sigma)$ is prime for all $\sigma$.

Definition sc.1. Suppose $\Delta$ is prime. Then the canonical model $\mathfrak{M}(\Delta)$ for $\Delta$ is defined by:

1. $W = \mathbb{N}^*$, the set of finite sequences of natural numbers.
2. $R$ is the partial order according to which $R\sigma\sigma'$ iff $\sigma$ is an initial segment of $\sigma'$ (i.e., $\sigma' = \sigma \bowtie \sigma''$ for some sequence $\sigma''$).
3. $V(p) = \{\sigma : p \in \Delta(\sigma)\}$.

It is easy to verify that $R$ is indeed a partial order. Also, the monotonicity condition on $V$ is satisfied. Since $\Delta(\sigma) \subseteq \Delta(\sigma.n)$ we get $\Delta(\sigma) \subseteq \Delta(\sigma')$ whenever $R\sigma\sigma'$ by induction on $\sigma$.

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Bibliography