sem.1 Semantic Notions

Definition sem.1. We say $\varphi$ is true in the model $M = \langle W, R, V \rangle$, $M \models \varphi$, iff $M, w \models \varphi$ for all $w \in W$. $\varphi$ is valid, $\models \varphi$, iff it is true in all models. We say a set of formulas $\Gamma$ entails $\varphi$, $\Gamma \vdash \varphi$, iff for every model $M$ and every $w$ such that $M, w \models \Gamma$, $M, w \models \varphi$.

Proposition sem.2.

1. If $M, w \models \Gamma$ and $\Gamma \vdash \varphi$, then $M, w \models \varphi$.
2. If $M \models \Gamma$ and $\Gamma \vdash \varphi$, then $M \models \varphi$.

Proof. 1. Suppose $M \models \Gamma$. Since $\Gamma \vdash \varphi$, we know that if $M, w \models \Gamma$, then $M, w \models \varphi$. Since $M, u \models \Gamma$ for all every $u \in W$, $M, w \models \Gamma$. Hence $M, w \models \varphi$.

2. Follows immediately from (1).

Definition sem.3. Suppose $M$ is a relational model and $w \in W$. The restriction $M_w = \langle W_w, R_w, V_w \rangle$ of $M$ to $w$ is given by:

- $W_w = \{ u \in W : Rwu \}$,
- $R_w = R \cap (W_w)^2$, and
- $V_w(p) = V(p) \cap W_w$.

Proposition sem.4. $M, w \models \varphi$ iff $M_w \models \varphi$.

Problem sem.1. Prove Proposition sem.4.

Proposition sem.5. Suppose for every model $M$ such that $M \models \Gamma$, $M \models \varphi$. Then $\Gamma \models \varphi$.

Proof. Suppose that $M, w \models \Gamma$. By the Proposition sem.4 applied to every $\psi \in \Gamma$, we have $M_w \models \Gamma$. By the assumption, we have $M_w \models \varphi$. By Proposition sem.4 again, we get $M, w \models \varphi$.

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Bibliography