sem.1  Semantic Notions

**Definition sem.1.** We say $\varphi$ is true in the model $\mathcal{M} = \langle W, R, V \rangle$, $\mathcal{M} \vDash \varphi$, iff $\mathcal{M}, w \vDash \varphi$ for all $w \in W$. $\varphi$ is valid, $\vDash \varphi$, iff it is true in all models. We say a set of formulas $\Gamma$ entails $\varphi$, $\Gamma \vDash \varphi$, iff for every model $\mathcal{M}$ and every $w$ such that $\mathcal{M}, w \vDash \Gamma$, $\mathcal{M}, w \vDash \varphi$.

**Proposition sem.2.**

1. If $\mathcal{M}, w \vDash \Gamma$ and $\Gamma \vDash \varphi$, then $\mathcal{M}, w \vDash \varphi$.
2. If $\mathcal{M} \vDash \Gamma$ and $\Gamma \vDash \varphi$, then $\mathcal{M} \vDash \varphi$.

**Proof.**

1. Suppose $\mathcal{M} \vDash \Gamma$. Since $\Gamma \vDash \varphi$, we know that if $\mathcal{M}, w \vDash \Gamma$, then $\mathcal{M}, w \vDash \varphi$. Since $\mathcal{M}, u \vDash \Gamma$ for all every $u \in W$, $\mathcal{M}, w \vDash \varphi$. Hence $\mathcal{M}, w \vDash \varphi$.

2. Follows immediately from (1).

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Bibliography