sem.1 Semantic Notions

int:sem:sem: sec

Definition sem.1. We say φ is *true in the model* $\mathfrak{M} = \langle W, R, V, w_0 \rangle$, $\mathfrak{M} \Vdash \varphi$, iff $\mathfrak{M}, w \Vdash \varphi$ for all $w \in W$. φ is *valid*, $\vDash \varphi$, iff it is true in all models. We say a set of formulas Γ *entails* φ, $\Gamma \vDash \varphi$, iff for every model $\mathfrak{M}$ and every w such that $\mathfrak{M}, w \Vdash \Gamma$, $\mathfrak{M}, w \Vdash \varphi$.

int:sem:sem: prop:sat-entails

Proposition sem.2.

int:sem:sem: prop:sat-entails1

1. *If $\mathfrak{M}, w \Vdash \Gamma$ and $\Gamma \vDash \varphi$, then $\mathfrak{M}, w \Vdash \varphi$.*

int:sem:sem: prop:sat-entails2

2. *If $\mathfrak{M} \Vdash \Gamma$ and $\Gamma \vDash \varphi$, then $\mathfrak{M} \Vdash \varphi$.*

Proof.

1. Suppose $\mathfrak{M} \Vdash \Gamma$. Since $\Gamma \vDash \varphi$, we know that if $\mathfrak{M}, w \Vdash \Gamma$, then $\mathfrak{M}, w \Vdash \varphi$. Since $\mathfrak{M}, u \Vdash \Gamma$ for all every $u \in W$, $\mathfrak{M}, w \Vdash \Gamma$. Hence $\mathfrak{M}, w \Vdash \varphi$.
2. Follows immediately from (1).

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Bibliography